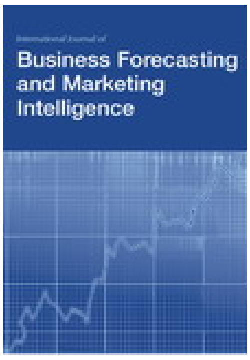

International Journal of Business Forecasting and Marketing Intelligence

ISSN online: 1744-6643 - ISSN print: 1744-6635
https://www.inderscience.com/ijbfmi

Handling the inconsistency in ranking customers via several multi-criteria ranking methods

Yossi Hadad, Baruch Keren, Dima Alberg

DOI: 10.1504/IJBFMI.2023.10055020

Article History:

Received:	21 February 2023
Last revised:	23 February 2023
Accepted:	28 February 2023
Published online:	03 January 2024

Handling the inconsistency in ranking customers via several multi-criteria ranking methods

Yossi Hadad, Baruch Keren* and Dima Alberg

SCE – Shamoon College of Engineering,
Beer Sheva, Israel
Email: yossi@sce.ac.il
Email: baruchke@sce.ac.il
Email: dimitria@sce.ac.il
*Corresponding author

Abstract: There are many multi-criteria methods for ranking and classifying customers, but it is difficult to determine which, if any, ranking method is the best. In this paper, we propose an application that uses several ranking methods, is easy to implement and retrieves ranking criteria values from the customer relationship management system. Because each ranking method can yield a different ranking for each customer, we suggest giving each ranking method the same weight so that each customer's final ranking will be determined by the average of the ranks obtained from all the ranking methods. A unique result of the proposed application is the possibility of calculating the variance of the rank for each customer and the confidence intervals. The applicability of the proposed method was demonstrated in a real case study with nine ranking methods; IPython codes of five of those methods are available herein.

Keywords: marketing intelligence; customer relationship management; CRM; multiple criteria decision analysis; MCDA; data envelopment analyses; DEAs; analytic hierarchy process; AHP; shortest distance ranking methods.

Reference to this paper should be made as follows: Hadad, Y., Keren, B. and Alberg, D. (2024) 'Handling the inconsistency in ranking customers via several multi-criteria ranking methods', *Int. J. Business Forecasting and Marketing Intelligence*, Vol. 9, No. 1, pp.50–79.

Biographical notes: Yossi Hadad is an Associate Professor and the Head of the Department of Industrial Engineering and Management at SCE – Shamoonn College of Engineering. His areas of specialty are DEA, work-study and ranking methods. He has published six books and about 50 papers in these areas. He received his BSc, MSc, and PhD (summa cum laude) in Industrial Engineering from the Ben-Gurion University of the Negev, Israel.

Baruch Keren is an Associate Professor in the Industrial Engineering and Management Department at the SCE – Shamoon College of Engineering and a Lecturer at the Open University of Israel. He received his BSc, MSc and PhD (Summa Cum Laude) in Industrial Engineering from the Ben-Gurion University of the Negev. His professional experience includes 13 years in Israel Chemicals Ltd. and its subsidiaries in the areas of industrial engineering, economics and auditing. His current research interests are decision-making under uncertainty, operations management, project management and operations research.

Dima Alberg is a Senior Lecturer in the Industrial Engineering and Management Department at the SCE – Shamoon College of Engineering. He received his PhD in Software and Information System Engineering Department from the Ben-Gurion University of the Negev. His current research interests are decision-making under uncertainty, finance data mining and data science.

1 Introduction

Customers are important business enterprise assets; they should be valued and managed (Gupta and Lehmann, 2003). Many companies value customers as part of their management activities to obtain the full benefit from their customers. Attracting customers, retaining existing customers, and satisfying customers by meeting their needs have become a challenge for many organisations (Keramati et al., 2012). Customers are usually different, so when customers are assessed, marketers look to Pareto Law which states that 80% of the profit is obtained from the 20% of the customers and 80% of the expenses are produced by the 20% of problematic customers (Kim et al., 2006).

Customers must be ranked and segmented to identify the best and worst among them. The difficulty is to find an appropriate ranking method that will distinguish the customers from each other, especially because customer ranking should be done according to multi-criteria ranking methods. In the literature, there are many ranking methods in which the relative scores of the customers can be calculated and, accordingly, the customers can be ranked. For example, an overview of customer ranking methods and their division into segments can be found in Hadad and Keren (2022). Beyond the difficulty in selecting the appropriate criteria (inputs and outputs) for ranking, each ranking method can yield different scores for each customer. As a result, each method can also yield a different ranking of the customers. As shown herein, some customers were ranked differently by each ranking method. This phenomenon will be presented later in a case study where customers were ranked according to nine different ranking methods.

This paper proposes a ranking procedure to help managers rank their customers. Moreover, if a firm already applies a specific ranking method, the proposed procedure evaluates the current ranking method's quality and reliability. The proposed procedure is a solution for the problem of inconsistency in ranking customers via several multi-criteria ranking methods. As mentioned, there are many methods for ranking and classifying customers, each has advantages and drawbacks, and each may yield a different rank, so it is difficult for decision-makers to adopt one agreed-upon ranking method. For example, ranking methods based on data mining and artificial intelligence require a large database with many customers and considerable activity (Nazari et al., 2020). Other ranking methods require unique software and a professional team to analyse the outputs and update the method from time to time for their development and ongoing operation, tasks that can be complex and expensive for small and medium-sized enterprises.

This paper presents several common multi-criteria ranking methods that are simple to implement, need only basic software tools such as Excel, and retrieve the values of the criteria for the ranking directly from the customer relationship management (CRM) system. Since each ranking method can yield a different ranking for each customer, our solution is to give each ranking method the same weight. With this equal-weight ranking method, the final ranking of each customer will be the average of the ranks that the customer obtained from all the selected ranking methods. Equal-weight method ranking also enables us to calculate the standard deviation and the confidence intervals of the rank of each customer. The confidence interval makes it possible to examine statistically the reliability of the ranks and the subjective assessments given to customers by the marketing team.

Ranking customers using only one ranking method may cause some customers to be incorrectly ranked. Therefore, we think using several well-known and proven methods for customer ranking is the preferred solution. This approach reduces the risk of a very wrong rank to some customers due to applying one inappropriate ranking method. The proposed procedure suggests using several ranking methods (at least five), relatively simple and easy to implement, where the data for their operation (inputs and outputs-criteria) is taken from the CRM system. After calculating the customers' scores of one ranking method, the rank of each customer according to this ranking method is determined. A similar procedure is applied to all the ranking methods, so at the end, each customer will have several ranks, one of each ranking method. The average of the ranks will determine the customer's final rank in each ranking method. Moreover, we also propose considering a different relative weight to each ranking method, based on the correlation coefficients among the results of all the ranking methods. Because each customer has several ranks, each obtained from another ranking method, one can calculate the standard deviation and the confidence interval of the average rank of each customer as a measure of the confidence of the final rank.

The rest of the paper is as follows: Section 2 presents a literature review; Section 3 describes the proposed ranking method; Section 4 presents in detail the ranking methods that were used; Section 5 is a case study; Section 6 concludes the paper.

2 Literature review

Companies are changing their business strategies from product-oriented strategies to customer-oriented strategies. Allotting marketing resources to customers optimally is necessary to rank the customers and find the best and the worst (Noorizadeh et al., 2013). This optimal distribution of marketing resources allows organisations to achieve a higher return on invested capital, gain more customers and take advantage of opportunities by understanding customer needs (Nikumanesh and Albadvi, 2014).

An essential instrument for customer control is the organisation's CRM system, a robust tool for managing the customers' data and supporting effective customer interaction. A CRM system can improve the overall performance of an organisation and is essential to its success, especially with significant changes in customer behaviours (Vogt, 2011). The CRM system is mainly used to optimise customer loyalty and extend the customer life cycle (El Essawi and El Aziz, 2012). A CRM system monitors an organisation's relationship with its customers to guarantee that the business activity with the customer is organised, efficient, synchronised, and productive (Lambert and Enz, 2017).

Many companies and organisations provide services and products to customers at various levels. Making a difference for different customers using a ranking approach is done to achieve different goals (Sajjadi et al., 2015). Customer ranking and segmentation are increasingly significant issues in the competitive commercial field (Hruschka, 1986). Customer evaluation and ranking have several purposes. Hadad and Keren (2022) surveyed a group of managers from different industrial organisations. Responses from the managers showed overlapping, which indicates that most had multiple concerns. For example, 87% of the managers stated that the goal of customer ranking is to identify problematic customers in order to improve their performance or get rid of them. Expressing a different focus, 31% of the managers stated that the goal is to identify the best customers in order to retain them. Viewing the CRM system as a whole, 15% stated that a customer-ranking model must allow examination of the correlation between the ranking and the level of satisfaction of each customer.

Customer ranking according to several criteria (outputs and inputs) usually results in a multiple criteria decision analysis (MCDA) or a multiple criteria decision making (MCDM). The use of MCDA focuses on designing mathematical and computational tools to support the subjective evaluation of a finite number of alternatives under a finite number of performance criteria. In an MCDM problem, a variety of alternatives (in our case, customers) are evaluated according to several criteria that characterise these alternatives to choose the best alternative (Rezaei, 2016).

Determining the appropriate criteria for ranking alternatives and classifications of each criterion as input or output is an important issue for MDCA. The selection of a different set of criteria may lead to different rankings and evaluations of the same customer. Widely applied criteria for customer evaluation are recency, frequency, and monetary (RFM), which have been used in many practical areas, particularly in marketing. By adopting an RFM model, decision-makers can effectively identify valuable customers and develop an effective marketing strategy. For more details about RFM models, see the survey by Jo-Ting et al. (2010).

Another critical issue for evaluating alternatives is determining the relative weight of each criterion. Different decision-makers may evaluate the importance (weight) of the criteria in different ways. Thus, the best alternative is subject to decision-makers' preferences (Mi et al., 2019). Pamučar et al. (2018) point out that determining criteria weights is one of the main problems of multi-criteria analysis models. Choosing an appropriate method for determining criteria weights is extremely important because the weights greatly influence the results (customer ranking in our case). The criteria weights depend significantly on the method used to set the weights. There is no agreement about the best method for determining criteria weights, and there is no agreement on how to determine the 'correct' set of weights.

Zavadskas et al. (2016) point out that the models for determining the criteria weights can be classified into subjective and objective models. Subjective approaches reflect the subjective opinion and intuition of the decision-maker. With such an approach, the decision-maker directly influences the results of the decision-making process. This influence is inescapable because the criteria weights are determined based on the information received from the decision-maker or the experts involved in the decision-making process. Objective approaches focus on determining the weight of criteria based on the information in a decision-making matrix that applies certain mathematical models. However, there is an understanding that weights calculated by applying certain methods are more accurate than the weights obtained by the judgment of experts (Pamučar et al., 2018). One of the methods for determining the relative weight is based on pairwise comparison. A pairwise comparison is a base of the analytic hierarchy process (AHP) (Saaty, 1980), a widely-applied method for ranking alternatives. In this method, decision-makers compare each criterion with the other criteria to determine the preference level for each pair.

Methods incorporating MCDA have received much attention from researchers and practitioners in evaluating, assessing, and ranking alternatives across diverse industries (Behzadian et al., 2012). Numerous methods have been proposed for ranking alternatives according to multiple criteria. For example, Velasquez and Hester (2013) surveyed a number of MCDM methods, including:

1. multi-attribute utility theory
2. AHP
3. fuzzy set theory
4. case-based reasoning
5. data envelopment analysis (DEA)
6. simple multi-attribute ranking technique
7. goal programming
8. ELECTRE
9. PROMETHEE
10. simple additive weighting
11. technique for order of preference by similarity to ideal solution.

The availability of many different MCDA methods emphasises the major problem of the MCDA: each method may produce different results for the same problem and the same dataset (Hadad and Hanani, 2011; Karim and Karmaker, 2016). Furthermore, in many ranking methods, adding or removing a customer from the evaluation group can change the internal rank of the customers. This phenomenon is called ‘a preference reversal’ (Lichtenstein and Slovic, 1971, 1973).

Ranking methods can also be classified into the following types:

1. Methods in the DEA context with common or non-common weights. In these ranking methods, the weight of each criterion is determined objectively (mathematically). A survey of these ranking methods can be found in Hadad and Hanani (2011) and Adler et al. (2002).
2. Subjective methods in which the weight of each criterion is determined subjectively by the decision-makers. The most known method is the AHP. A survey of subjective ranking methods can be found in Hadad and Hanani (2011).
3. Methods based on the shortest distance from the best or the ideal solution. These methods indicate the best alternative with the shortest distance from the positive ideal solution and the farthest from the negative ideal solution (Benitez et al., 2007). For example, a positive ideal solution maximises the profit criteria and minimises the cost criteria, while a negative ideal solution minimises the profit criteria and maximises the cost criteria (Karim and Karmaker, 2016).
4. Ranking methods that combine several different ranking methods. For example, a model that combines AHP and DEA (Sinuany-Stern et al., 2000) and the technique

for order preference by similarity to ideal solution (TOPSIS), developed by Yoon and Hwang (1985) and improved by Singaravel and Selvaraj (2015) is a model that combines AHP and the shortest distance concept. This paper will present five methods that combine shortest distance-based methods and AHP.

DEA is one of the most common methods used to determine the efficiency of decision-making units (Charnes et al., 1978). The DEA is a systematic approach where the criteria weights are determined objectively as a part of the method. The DEA method converts a nonlinear ratio measure of efficiency into a linear programming problem. As a result, decision-making units (in our case customers) could be assessed on the basis of multiple inputs and outputs, even if the production function is unknown. A focused DEA can be used to evaluate the performance of customers and for ranking (Izadikhah and Farzipoor Saen, 2020). The application of the DEA method enables each customer to have its own production function and then to estimate the efficiency of that individual customer by comparing it with the efficiency of the other customers in the dataset. The DEA classifies the customers into two groups: efficient, with an efficiency score of 100%, and inefficient, with an efficiency score of less than 100%. This classification separation is a strength and a weakness of the standard DEA model because, while it lets DEA assess the efficiency of each dataset, it does not have the power to obtain a full ranking of all the customers (Aldamak and Zolfaghari, 2017). Suppose the decision-makers are interested in a full ranking of the customers, beyond the dichotomous classification (efficient and inefficient). In that case, the standard DEA method is not applicable, and ranking methods must be used (Adler et al., 2002).

The interest in customer evaluation, segmentation, and ranking has remained high in the scientific literature. By using new and updated methods for customer evaluation, marketing managers can more effectively target the most valuable customers, and reduce the costs due to wrongly targeted valuable clients. Here are several examples of these topics. Christy et al. (2021) performed an RFM analysis on the customer data and then extends it to clusters using traditional K-means and fuzzy C-means algorithms, with a novel idea for choosing the initial centroids in K-means. Djurisic et al. (2020) proposed a predictive approach to segmenting credit card users, based on their value to the bank. Their approach combines RFM, clustering using the K-means method, and predictive classification by the support vector machine method. Rogić and Kašćelan (2021) proposed a class-balancing approach based on support vector machine-rule extraction and ensemble learning. Their approach allows for rule extraction, which can describe and explain different customer segments. Nie et al. (2021) developed a method to classify customers according to their value to an organisation. The initial step of their method is to construct a full customer history and extract a feature set suited to customer lifetime value calculations. Singh et al. (2020) proposed customer segmentation based on demographic properties like gender, age, and spending score and analysed the dataset for interesting facts. The derived attribute dataset was used to classify each customer into several classes. Alaswad et al. (2021) categorised customers based on their buying patterns taking into account the loyalty score, which was calculated using the RFM model. After performing the categorisation process, several models are trained and used to predict the cluster of customers.

In the next section, we will present our proposed ranking model based on scores obtained from several known ranking methods. After that, we will survey nine known objective ranking methods, simple and easy to implement, used in our model to rank the customers. These nine methods were adjusted to the scenario of customer ranking.

3 The proposed model

In this section, we present the proposed model for ranking customers. The model is based on several multi-criteria ranking methods that objectively calculate customers' relative scores. Ranking the customers according to the scores of one ranking method generates a full rank of the customers according to this ranking method. The value of each criterion of each customer can be retrieved from the CRM system with, perhaps, some necessary calculations. The criteria values should be retrieved and calculated periodically, at the appropriate frequencies, taking into account changes in the business environment and customers' behaviour. Periodically running the model enables the examination of changes in the relative ranking and evaluation of the results of organisational efforts given to the customers.

3.1 The steps of the proposed model

The steps of the proposed mode are executed as follows:

Step 1 Define the period for collecting value criteria (month, quarter, and year) and define the customers to be ranked. The selection of the customers to be evaluated and ranked is significant because adding or removing a customer can change the scores of other customers and their internal ranking. This phenomenon of rank reversal exists in many multi-criteria ranking methods. A survey about rank reversal can be found in a review paper by Aires and Ferreira (2018).

Step 2 Define the criteria for ranking the customers. Choose objective criteria whose values can be retrieved from the CRM system. Classify the criteria into two groups, outputs and inputs. The criteria should not exceed 1/3 of the number of customers (Banker et al., 1984).

Step 3 Choose several ranking methods for evaluating and ranking the customers. We recommend selecting simple, objective, and known ranking methods that can be applied in any enterprise. Later in this section, we will present several ranking methods we applied in our case study.

Step 4 According to the first selected ranking method, $l = 1$ calculate the score of each customer $S_{1,j}$ $j = 1, \ldots, n$. Then, rank all the customers in descending order according to these scores (where customer K with the maximum score $\underset{j}{Max}\{S_{1,j}\}$ will be ranked first ($R_{1,K} = 1$), and customer T with the minimum score $\underset{j}{Min}\{S_{1,j}\}$ will be ranked last ($R_{1,T} = n$). Do this step for all the selected ranking methods l ($l = 1, \ldots, L$). At the end of this step, each customer will have l rank values $R_{1,j}, \ldots, R_{L,j}$.

Step 5 Calculate the average ranking of each customer [equation (1)]

$$\overline{R}_j = \frac{\sum_{l=1}^{L} R_{l,j}}{L} \quad j = 1, 2, \ldots, n. \tag{1}$$

Then, rank all the customers according to their average ranking in increasing order (where customer K with the minimum average ranking $\underset{j}{Min}\{\overline{R}_j\}$ will be ranked first ($R_K = 1$), and customer T with the maximum average ranking $\underset{j}{Max}\{\overline{R}_j\}$ will be ranked last ($R_K = n$).

Step 6 Calculate the confidence interval of the average rank $\overline{R}_j$ for all customers based on the L values of the rank $R_{1,j}, \ldots, R_{L,j}$ of each customer. After calculating the average score $\overline{R}_j$ and the standard deviation σ_j by the L values of customers' rank, j, the confidence interval is calculated according to equation (2).

$$\overline{R}_j - \frac{t_{\alpha/2} \times \sigma_j}{\sqrt{L}} < R_j < \overline{R}_j + \frac{t_{\alpha/2} \times \sigma_j}{\sqrt{L}}, \tag{2}$$

where $t_{\alpha/2}$ is the value of the t distribution with $v = L - 1$ degrees of freedom, leaving an area $\alpha/2$ to the right. The confidence interval gives lower and upper bounds for each customer's rank range.

Another proposed possibility for ranking the customers is using a weighted average instead of a simple one. In this possibility, the rank of each customer will be calculated as shown in equation (3),

$$\overline{R}_j = \sum_{l=1}^{L} W_l \times R_{l,j} \quad j = 1, 2, \ldots, n, \tag{3}$$

where W_l is the weight of the ranking method l.

Denote the correlation coefficient between the ranking method and the ranking method. We propose that the weights will be calculated according to the correlation coefficient among the ranking methods. The weight Wl is calculated by equation (4) as follows:

$$W_l = \frac{\sum_{l \neq k} \rho_{l,k}}{\sum_{l=1}^{L} \sum_{l \neq k} \rho_{l,k}} \quad l = 1, \ldots, L. \tag{4}$$

The assumption is that a ranking method with a low sum of correlation coefficients has biased results compared to the other ranking methods. Therefore, giving a ranking method with extreme results a lower weight in determining the final ranking may be better.

4 Ranking methods that were used

In this subsection, we will present several common ranking methods that we will use for customer ranking in our case study. These ranking methods can be classified into two types: ranking methods in the context of the DEA [super efficiency (SE), discriminant DEA (DR/DEA) of ratios, canonical correlation analysis (CCA), global efficiency (GE) and ranking methods that are based on the shortest distance from a best or an ideal solution]. These shortest-distance methods use the AHP process to evaluate distances from an average solution. These methods, that almost all of which include AHP as part of their abbreviation, include the TOPSIS, the multi-criteria optimisation and compromise solution (VIKOR), the complex proportional assessment (COPRAS) method, the stable preference ordering towards ideal solution SPOTIS method, and the evaluation based on distance from average solution (EDAS) method.

4.1 Super efficiency

Consider n customers that should be ranked, where each customer is characterised by multiple inputs and outputs (criteria). Each customer j ($j = 1, 2, \ldots, n$) uses m types of inputs $X_j = (x_{1,j}, \ldots, x_{m,j}) > 0$ that are needed for producing s types of outputs $Y_j = (y_{1,j}, y_{2,j}, \ldots, y_{s,j}) > 0$. The relative efficiency of customer k is defined as the ratio of the total weighted output to the total weighted input, as given in equation (5).

$$h_k = \frac{\sum_{r=1}^{s} u_r^k y_{r,k}}{\sum_{i=1}^{m} v_i^k x_{i,k}} \tag{5}$$

The weights u_r^k ($r = 1, 2, \ldots, s$) and v_i^k ($i = 1, 2, \ldots, m$), are non-negative. The weights are calculated for each customer, separately, so each customer has individual weights for the criteria. For each customer k, the DEA calculates the optimal weight for each input v_i^k ($i = 1, 2, \ldots, m$) and each output u_r^k ($r = 1, 2, \ldots, s$), to maximise relative efficiency h_k, subject to that $h_j \le 1$, $j = 1, \ldots, n$. Equation (5) can be translated into a linear programming problem by adding the constraint $\sum_{i=1}^{m} v_i^k x_{i,k} = 1,$ as proposed by Charnes et al. (1978), which can be easily solved. The weights are calculated separately for each customer, so each customer has individual weights for the criteria.

One of the drawbacks of DEA is that it does not rank the efficient customers because they all have scores of 1. To overcome this drawback, Andersen and Petersen (1993) suggested allowing efficient customers to receive a score greater than one by dropping the constraint that bounds the score of the evaluated customer to 1. The linear programming problem of Andersen and Petersen for calculating the score S_k of customer k is formulated as follows:

$$S_k = \max \sum_{r=1}^{s} u_r^k y_{r,k}$$
$$\text{Subject to:}$$
$$\sum_{i=1}^{m} v_i^k x_{r,k} = 1 \tag{6}$$
$$\sum_{r=1}^{s} u_r^k y_{r,j} - \sum_{i=1}^{m} v_i^k x_{r,j} \le 0 \text{ for } j = 1, 2, \ldots, n \ \ j \ne k$$
$$u_r^k \ge \varepsilon \text{ for } r = 1, 2, \ldots, s.$$
$$v_i^k \ge \varepsilon \text{ for } i = 1, 2, \ldots, m.$$

The value of ε is a specific bound on the weights.

4.2 DR/DEA of ratios

Sinuany-Stern and Friedman (1988) developed a method that provides the best common weights for given inputs and outputs for all the customers. Their method discriminates optimally between efficient and inefficient customers as obtained by the DEA, which enables ranking all the customers on the same scale. This method, called DR/DEA, works through DR/DEA of ratios. The procedure begins with DEA, which generates two groups, efficient and inefficient customers. For each customer, the ratio $T_j = \sum_{r=1}^{s} u_r \times Y_{r,j} \Big/ \sum_{i=1}^{m} v_i \times X_{i,j}$ is calculated with arbitrary initial weights. Then, the arithmetic means of the ratio score of the efficient group $(\overline{T}_1)$ and inefficient group $(\overline{T}_2)$ are calculated $\overline{T}_1 = \sum_{j=1}^{n_1} T_j \Big/ n_1$, $\overline{T}_2 = \sum_{j=n_1+1}^{n} T_j \Big/ n$, where n_1 and n_2 are the respective numbers of efficient and inefficient customers in the DEA model.

The between-group variance $SS_B(T)$ is calculated as follows: $SS_B(T) = n_1(\overline{T}_1 - \overline{T})^2 + n_2(\overline{T}_2 - \overline{T})^2$, and the within-group variance $SS_W(T)$ is calculated as follows: $SS_W(T) = \sum_{j=1}^{n_1}(T_j - \overline{T}_1)^2 + \sum_{j=n_1+1}^{n_2}(T_j - \overline{T}_2)^2$. The objective function is to find common weights, as shown in equation (7), u_r ($r = 1, 2, \ldots, s$) and v_i ($i = 1, 2, \ldots, m$) that maximise the ratio between $SS_B(T)$ and $SS_W(T)$, namely:

$$\max_{u_r, v_i} \lambda = \frac{SS_B(T)}{SS_W(T)}, \tag{7}$$

and customer j's score is calculated as shown in equation (8):

$$S_j = \frac{\sum_{r=1}^{s} u_r \times Y_{r,j}}{\sum_{i=1}^{m} v_i \times X_{i,j}}, \quad j = 1, \ldots, n. \tag{8}$$

For more details, see Sinuany-Stern and Friedman (1988).

4.3 Canonical correlation analysis

The CCA determines two vectors of coefficients, weights of inputs v_i (i = 1, 2, …, m), and weights of outputs u_r (r = 1, 2, …, s). The values of the weights maximise the correlation between the weighted output $O_j = \sum_{r=1}^{s} u_r \times Y_{r,j}$, and the weighted input $I_j = \sum_{i=1}^{m} v_i \times X_{i,j}$ over all the customers (for more details, see Tatsuoka and Lohnes, 1988). Friedman and Sinuany-Stern (1997) developed the CCA into a ranking method. Their idea was that after obtaining the weights that maximise the correlation, customer j's score would be calculated by the same concept of equation (8).

4.4 Global efficiency

Ganley and Cubbin (1992) developed GE, a method that provides the best common weights for all the given inputs and outputs criteria. The weights of inputs v_i (i = 1, 2, …, m) and outputs u_r (r = 1, 2, …, s) are common weights that maximise the sum $\sum_{j=1}^{n} S_j$, calculated by the following optimisation problem presented as Equation (9).

$$
\begin{aligned}
&\max \sum_{j=1}^{n} S_j = \sum_{j=1}^{n} \frac{\sum_{r=1}^{s} u_r \times Y_{r,j}}{\sum_{i=1}^{m} v_i \times X_{i,j}} \\
&\text{s.t.} \\
&S_j \le 1 \ \ j = 1, 2, \ldots, n. \\
&u_r \ge \varepsilon \text{ for } r = 1, 2, \ldots, s. \\
&v_i \ge \varepsilon \text{ for } i = 1, 2, \ldots, m.
\end{aligned}
\tag{9}
$$

The optimal S_j obtained by equation (9) is the score of customer j.

4.5 Technique for order preference by similarity to ideal solution

The TOPSIS was developed by Yoon and Hwang (1985). This technique determines the best alternative that simultaneously has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The TOPSIS method is one of the most widely used multi-criteria decision analysis methods, as discussed, for example, by Behzadian et al. (2012), Ferreira et al. (2018) and de Farias Aires and Ferreira (2019). Our TOPSIS version, adjusted for customer ranking, is determined through the following procedure:

Step 1 Construct the criteria matrix P.

Let $x_{i,j}$ the value of input i ($i = 1, \ldots, m$) and $y_{r,j}$ the value of output r ($r = 1, \ldots, s$) both of customer j ($j = 1, \ldots, n$). Let us define an input-output values matrix P of all the criteria, for all the customers, with the element $P_{j,k}$ $j = 1, \ldots, n$, $k = 1, \ldots, m + s$, as shown in equation (10).

$$P = \left[p_{j,k}\right]_{n\times(m+s)} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,m} & \cdots & p_{1,m+s} \\ p_{2,1} & \cdots & p_{2,m} & \cdots & p_{2,m+s} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n,1} & \cdots & p_{n,m} & \cdots & p_{n,m+s} \end{bmatrix} \tag{10}$$

Step 2 Normalise the matrix P and construct the matrix Q.

An element $q_{j,k}$ of matrix Q is calculated according to equation (11).

$$q_{j,k} = \frac{p_{j,k}}{\sqrt{\sum_{k=1}^{m+s}\sum_{j=1}^{n} p_{j,k}^2}}, \tag{11}$$

Step 3 Evaluate the weight of each criterion, (w_k $k = 1, \ldots, m + s$).

To use this ranking method, the weight of each criterion must be determined in advance. Several approaches can determine the weights. For example,

1 the same weight to all the criteria

2 weights that are determined subjectively by decision-makers

3 weights that are determined according to AHP.

This paper used the AHP approach for all the shortest-distance-based methods.

Step 4 Construct the weighted normalised matrix L by multiplying the weights.

An element of the matrix L is calculated according to equation (12).

$$l_{j,k} = w_k \times q_{j,k} \quad j = 1, \ldots, n,\ k = 1, \ldots, m+s. \tag{12}$$

Step 5 Determine the positive and negative solutions for each input and output, according to equation (13).

$$\begin{aligned} &A^+ = \left\{l_1^+, \ldots, l_{m+s}^+\right\} \text{ where} \\ &l_k^{+*} = \left\{\max\left(l_{j,k}\right) \text{ if } k \text{ is outputs; } \min\left(l_{j,k}\right) \text{ if } k \text{ is an input}\right\}, \\ &A^- = \left\{l_1^-, \ldots, l_{m+s}^-\right\} \text{ where} \\ &l_k^{-*} = \left\{\min\left(l_{j,k}\right) \text{ if } k \text{ is outputs; } \max\left(l_{j,k}\right) \text{ if } k \text{ is an input}\right\}. \end{aligned} \tag{13}$$

Step 6 Calculate each customer's separation measures (positive and negative) according to equation (14).

$$d_j^+ = \sqrt{\sum_{k=1}^{m+s}\left(l_k^{+*} - l_{j,k}\right)^2},\ d_j^- = \sqrt{\sum_{k=1}^{m+s}\left(l_k^{-*} - l_{j,k}\right)^2}\quad j = 1, \ldots, n. \tag{14}$$

Step 7 Calculate the relative closeness coefficient to the ideal solution of each customer according to equation (15).

$$S_j = \frac{d_j^-}{d_j^- + d_j^+}\quad j = 1, \ldots, n. \tag{15}$$

The value S_j obtained by equation (15) is customer j's score.

4.6 Multi-criteria optimisation and compromise solution

The VIKOR compromise-ranking algorithm, translated to English from the original Serbian (Opricovic, 1998; Opricovic and Tzeng, 2004), has the following steps:

Step 1 Use equation (16) to determine the best f_k^* and the worst f_k^- values of all the criterion functions, where $k = 1, \ldots, m + s$ are the criteria, and $j = 1, \ldots, n$ are the customers.

$$\begin{aligned} &f_k^* = \max_j\left\{p_{j,k}\right\},\ f_k^- = \min_j\left\{p_{j,k}\right\}, \text{if the } k^{\text{th}} \text{ function is an output;} \\ &f_k^* = \min_j\left\{p_{j,k}\right\},\ f_k^- = \max_j\left\{p_{j,k}\right\}, \text{if the } k^{\text{th}} \text{ function is an input.} \end{aligned} \tag{16}$$

Step 2 Evaluate each criterion's weight, with the weights' values calculated using AHP.

Step 3 Compute for each customer the values D_j and R_j, $j = 1, \ldots, n$, using the following equation (17):

$$\begin{aligned} &D_j = \sum_{k=1}^{m+s} w_k\left(f_k^* - p_{j,k}\right)/\left(f_k^* - f_k^-\right) \\ &R_j = \max_k\left[w_k\left(f_k^* - p_{j,k}\right)/\left(f_k^* - f_k^-\right)\right], \end{aligned} \tag{17}$$

Step 4 Compute the score S_j, $j = 1, \ldots, n$, of each customer by the following equation (18):

$$S_j = v\left(D_j - D^*\right)/\left(D^- - D^*\right) + (1 - v)\left(R_j - R^*\right)/\left(R^- - R^*\right), \tag{18}$$

where $D^* = \min_j\{D_j\}$, $D^- = \max_j\{D_j\}$, $R^* = \min_j\{R_j\}$, $R^- = \max_j\{R_j\}$, v is the weight for the strategy of maximum group utility, and $(1 - v)$ is the weight of the regret. Note that, generally, by consensus $v = 0.5$ (Opricovic and Tzeng, 2007).

The value S_j obtained by equation (18) is customer j's score.

4.7 Complex proportional assessment

The COPRAS method was introduced by Zavadskas et al. (1994) and included the following steps:

Step 1 Construct an input-output values matrix P as described in equation (10).

Step 2 Normalise matrix P. An element $\overline{p}_{j,k}$ is calculated according to equation (19).

$$\overline{p}_{j,k} = p_{j,k} \Big/ \sum_{k=1}^{m+s} p_{j,k} \quad j = 1, \ldots, n, \ k = 1, \ldots, m+s. \tag{19}$$

Step 3 Evaluate the weight of each criterion w_k $k = 1, \ldots, m + s$ (using AHP) and calculate the weighted normalised matrix $\overline{L}$ where weighted normalised values $\hat{p}_{j,k}$ are calculated according to equation (20).

$$\hat{p}_{j,k} = w_k \times \overline{p}_{j,k} \ \ j = 1, \ldots, n, \ k = 1, \ldots, m+s. \tag{20}$$

Step 4 Sum the normalised-weighted values of the outputs (where higher values are preferable) and name p_j for each customer j ($j = 1, \ldots, n$) [see equation (21)].

$$p_j = \sum_{j=1}^{n} \hat{p}_{j,k} \ \ k = m+1, \ldots, m+s. \tag{21}$$

Step 5 Sum the normalised-weighted values of the inputs (where lower values are preferable) and name R_j for each customer j ($j = 1, \ldots, n$) [see equation (22)].

$$R_j = \sum_{j=1}^{n} \hat{p}_{j,k} \ \ k = 1, \ldots, m. \tag{22}$$

Step 6 Calculate the relative weight for each customer according to equation (23).

$$Q_j = p_j + R_{\min} \sum_{j=1}^{n} R_j \Big/ R_{\min} \times R_j \sum_{j=1}^{n} (1/R_j) = p_j + \sum_{j=1}^{n} R_j \Big/ R_j \sum_{j=1}^{n} (1/R_j) \tag{23}$$

Step 7 Compute the score S_j, $j = 1, \ldots, n$, of each customer by equation (24):

$$S_j = \frac{Q_j}{\max_j \{Q_j\}} \ \ j = 1, \ldots, n. \tag{24}$$

The value S_j obtained by equation (24) is customer j's score.

4.8 Stable preference ordering toward the ideal solution

The SPOTIS ranking method was introduced by Dezert et al. (2020). The SPOTIS method exempts a rank reversal. The method begins with matrix P (9) and has the following steps:

Step 1 Define the minimum and maximum values for the original criteria (inputs and outputs) $p_k^{\max}$, $p_k^{\min}$ ($k = 1, \ldots, m + s$) The reasonable theoretical minimum and maximum bounds are evaluated by expert judgment in any actual application.

If a criterion k is an input, then the best ideal solution for this criterion k is $p_k^{\min}$, and if a criterion k is an output, then the best ideal solution is $p_k^{\max}$. Therefore, the theoretical optimal solution, as shown in equation (25), is:

$$p^* = \left(p_1^*, \ldots, p_{m+s}^*\right) = \left(p_1^{\min}, \ldots, p_m^{\min}, p_{m+1}^{\max}, \ldots, p_{m+s}^{\max}\right). \tag{25}$$

In our case study, the minimum (maximum) bound of an input (output) criterion was determined as the minimum (maximum) value found at one of the customers.

Step 2 Evaluate the weight of each criterion; the weights express the decision-makers' preference about the relative importance of the criteria or set the weights subjectively using AHP, as done in this paper.

Step 3 For each customer j, $j = 1, \ldots, n$, compute its normalised distance concerning the ideal solution S^* for each criterion k. This normalised distance of the performance of customer j, $j = 1, \ldots, n$, from the Ideal solution for criterion K, is calculated as shown in equation (26), is:

$$d_{j,k} = \frac{\left|p_{j,k} - p_k^*\right|}{\left|p_k^{\max} - p_k^{\min}\right|}. \tag{26}$$

Step 4 The normalised averaged distance concerning multi-criteria from the ideal solution (customer j's score) is computed as shown in equation (27):

$$S_j = \sum_{k=1}^{m+s} w_k \times d_{j,k} \quad j = 1, \ldots, n. \tag{27}$$

The value of S_j obtained by equation (27) is customer j's score.

4.9 Evaluation based on distance from average solution

The EDAS evaluation based on distance from the average solution method was introduced by Ghorabaee et al. (2015). The EDAS method has the following steps.

Step 1 Prepare decision-making matrix P as matrix (9).

Step 2 Determine the average solution according to all criteria, as shown in equation (28):

$$AV_k = {\sum_{j=1}^{n} p_{j,k}} \Big/ {n} \qquad k = 1, \ldots, m+s. \tag{28}$$

Step 3 Calculate the positive distance from average (PDA) and the negative distance from average (NDA) matrixes according to the type of criteria (output and input).

If the k^{th} criterion is output, complete the calculations as shown in equation (29):

$$PDA_{j,k} = \max\{0, (P_{j,k} - AV_k)\}/AV_k;$$
$$NDA_{j,k} = \max\{0, (AV_k - P_{k,j})\}/AV_k. \tag{29}$$

If the k^{th} criterion is input, complete the calculations as shown in equation (30):

$$PDA_{j,k} = \max\{0, (AV_k - p_{j,k})\}/AV_k;$$
$$NDA_{j,k} = \max\{0, (p_{j,k} - AV_k)\}/AV_k. \tag{30}$$

Note that $PDA_{j,k}$ and $NDA_{j,k}$ denote the positive and negative distance of the j^{th} customer from the average solution in terms of the k^{th} criterion.

Step 4 Evaluate the weight of each criterion, w_k k =1, …, $m + s$, set the weights subjectively using AHP, as done in this paper, and determine the weighted sum of *PDA* and *NDA* for all the customers as shown in equation (31):

$$SP_j = \sum_{k=1}^{m+s} w_k \times PDA_{j,k}; \; SN_j = \sum_{k=1}^{m+s} w_k \times NDA_{j,k}. \tag{31}$$

Step 5 Normalise the values of *SP* and *SN* for all the customers as shown in equation (32):

$$NSP_j = \frac{SP_j}{\max\{SP_j\}}; \; NSN_j = \frac{SN_j}{\max\{SN_j\}}. \tag{32}$$

Step 6 Compute the score S_j, j = 1, …, n, of each customer as shown in equation (33):

$$S_j = \frac{1}{2}(NSP_j + NSN_j) \; j = 1, \ldots, n. \tag{33}$$

The value of S_j obtained by equation (33) is customer j's score.

5 Case study

To illustrate the proposed method, we use the same data from the 43 customers presented by Hadad and Keren (2022). For the convenience of the readers, the tables of criteria and the values of these criteria (matrix *P*) are in Appendix B (Tables B1 and B2).

Applying DEA [Charnes et al. (1978) model] to outline the inputs and outputs given in matrix *P* (Table B2) yields the scores that are shown in Table 3. The results in Table 1 show that 22 customers are efficient (E_j = 1) and 21 are inefficient (E_j < 1).

The weights of the criteria for the ranking methods that need pre-determined weights (the shortest distance ranking methods), were determined by the AHP model. Two AHP pairwise comparison matrixes were set, one for input criteria and one for output criteria. The decision-makers of the plant where the case study was conducted determined the values of these matrixes. The inconsistencies of the evaluations in pairwise comparison matrixes were 6.24% for the input criteria and 4.64% for the output criteria. The obtained AHP weights are in Table 2.

Table 1 The DEA efficiency score of each customer

Customer	E_j
A1	1
A2	1
A3	0.1407
A4	0.9379
A5	0.8033
A6	1
A7	0.5560
A8	0.9974
A9	0.5391
A10	0.8123
A11	0.8559
A12	0.9496
A13	1
A14	0.4458
A15	0.5540
A16	1
A17	1
A18	1
A19	1
A20	0.4589
A21	1
A22	0.5046
A23	1
A24	1
A25	0.9509
A26	0.4502
A27	1
A28	0.5746
A29	1
A30	0.9773
A31	0.5567
A32	1
A33	1
A34	1
A35	0.0309
A36	1
A37	1
A38	1

Table 1 The DEA efficiency score of each customer (continued)

Customer	E_j
A39	1
A40	1
A41	1
A42	0.8869
A43	0.8429

Table 2 Weights of each criterion according to the AHP method

Input weights						*Output weights*			
W_{X_1}	W_{X_2}	W_{X_3}	W_{X_4}	W_{X_5}	W_{X_6}	W_{Y_1}	W_{Y_2}	W_{Y_3}	W_{Y_4}
0.0739	0.0262	0.0739	0.3038	0.2184	0.3038	0.2481	0.0896	0.0429	0.6194

Table 3 Scores of each customer using different ranking methods

Customer	*SE*	*DR/ DEA*	*CCA/ DEA*	*GE*	*TOPSIS*	*VICOR*	*COPRAS*	*SPOTIS*	*EDAS*
A1	1.3511	19.0388	0.3857	0.6721	0.6588	0.7274	0.6010	0.7612	0.6871
A2	2.0022	15.1962	0.3720	0.5735	0.7192	0.8312	0.7280	0.8591	0.7454
A3	0.1407	8.3756	0.2136	0.3158	0.4230	0.1368	0.3775	0.3762	0.3996
A4	0.9379	13.2282	0.4419	0.5863	0.4527	0.2648	0.4432	0.5157	0.5272
A5	0.8033	8.1419	0.3266	0.4171	0.4611	0.2309	0.4171	0.4459	0.4473
A6	1.5446	30.8235	0.7089	1.0000	0.6303	0.5511	0.6688	0.7721	0.6859
A7	0.5560	10.2217	0.3487	0.4125	0.5006	0.2628	0.4159	0.4772	0.4819
A8	0.9974	15.1259	0.3784	0.6371	0.6564	0.7126	0.5641	0.6978	0.6710
A9	0.5391	11.0556	0.2953	0.4577	0.5441	0.4349	0.4489	0.5044	0.5328
A10	0.8123	13.9495	0.2930	0.5321	0.5071	0.3416	0.4940	0.5901	0.5661
A11	0.8559	10.5596	0.2823	0.3353	0.5441	0.3808	0.4455	0.5311	0.5284
A12	0.9496	14.7875	0.3911	0.6007	0.7344	0.8497	0.6441	0.8057	0.7383
A13	3.5221	14.9868	0.5938	0.4934	0.3002	0.0198	0.5203	0.5000	0.2778
A14	0.4458	8.0129	0.1779	0.2921	0.4693	0.2515	0.4104	0.4446	0.4425
A15	0.5540	7.0746	0.2115	0.2992	0.3810	0.0749	0.3608	0.3627	0.3481
A16	3.2422	24.5639	1.0000	1.0000	0.7496	0.9017	1.0000	0.9442	0.7947
A17	1.7641	28.6005	0.9374	0.9753	0.4562	0.3064	0.5621	0.6562	0.5711
A18	2.0601	30.2138	1.0000	1.0000	0.5153	0.3607	0.6442	0.6986	0.5996
A19	1.4843	17.4449	0.1623	0.3907	0.4895	0.3520	0.5345	0.6453	0.5605
A20	0.4589	9.3863	0.2692	0.3734	0.5474	0.3661	0.6247	0.6776	0.5892
A21	2.5051	23.3622	0.7902	0.6981	0.4720	0.3198	0.4657	0.6149	0.5401
A22	0.5046	12.4349	0.1903	0.3427	0.5108	0.3273	0.5021	0.6200	0.5474
A23	8.2141	28.7071	0.6738	1.0000	0.7633	1.0000	0.8421	0.9382	0.8198
A24	1.4845	20.7567	0.4913	0.8691	0.4428	0.3175	0.5183	0.6419	0.5415
A25	0.9509	11.5381	0.2018	0.3538	0.4587	0.2545	0.4200	0.5299	0.4939

Table 3 Scores of each customer using different ranking methods (continued)

Customer	*SE*	*DR/ DEA*	*CCA/ DEA*	*GE*	*TOPSIS*	*VICOR*	*COPRAS*	*SPOTIS*	*EDAS*
A26	0.4502	10.5516	0.2650	0.3609	0.5558	0.4645	0.4509	0.4572	0.5106
A27	2.1739	18.1597	0.3160	0.4924	0.4963	0.3490	0.4655	0.5502	0.5481
A28	0.5746	13.4456	0.3439	0.4833	0.4310	0.1890	0.3995	0.4437	0.4827
A29	1.5382	15.2820	0.5075	0.6100	0.7187	0.8138	0.5864	0.7276	0.7265
A30	0.9773	10.2118	0.5188	0.6576	0.4196	0.1804	0.4112	0.4791	0.4514
A31	0.5567	12.1133	0.3242	0.4680	0.6087	0.5387	0.4862	0.5494	0.5910
A32	2.1878	9.5632	0.3298	0.4498	0.3517	0.1432	0.3712	0.4420	0.3968
A33	1.4484	23.3548	0.3983	0.5218	0.6315	0.5793	0.6683	0.7423	0.6893
A34	2.0738	25.2071	0.6170	1.0000	0.6137	0.5403	0.6602	0.7429	0.6742
A35	3.2087	20.1271	0.4407	0.7505	0.5950	0.5307	0.5901	0.7440	0.6366
A36	1.2553	17.0072	0.6097	0.8350	0.6883	0.7009	0.6081	0.7276	0.6855
A37	1.3222	16.1696	0.5040	0.8644	0.6179	0.5558	0.5804	0.7331	0.6367
A38	1.2208	20.3500	0.4784	0.4704	0.5836	0.4446	0.4987	0.6100	0.6247
A39	1.1037	13.8738	0.4150	0.4575	0.6547	0.6254	0.5120	0.6180	0.6308
A40	1.4804	17.1381	0.2720	0.5760	0.5842	0.5299	0.5374	0.6700	0.6086
A41	2.1756	16.5872	0.4735	1.0000	0.3579	0.2081	0.4185	0.5827	0.4079
A42	0.8869	15.7881	0.6037	0.6441	0.6200	0.5116	0.5670	0.6580	0.6431
A43	0.8429	7.8305	0.3049	0.3793	0.3321	0.1018	0.3364	0.4156	0.3713

Table 4 The rank of each customer using different ranking methods

Customer	*SE*	*DR/ DEA*	*CCA/ DEA*	*GE*	*TOPSIS*	*VICOR*	*COPRAS*	*SPOTIS*	*EDAS*
A1	19	12	22	13	7	6	11	6	7
A2	11	21	24	21	4	4	3	3	3
A3	43	39	38	41	37	40	40	42	39
A4	28	28	17	19	34	31	32	31	30
A5	33	40	28	32	31	35	35	37	36
A6	13	1	5	3	11	12	4	5	8
A7	36	35	25	33	26	32	36	35	34
A8	24	22	23	16	8	7	16	14	11
A9	38	32	32	29	21	20	30	32	28
A10	32	25	33	22	25	26	25	25	22
A11	30	33	34	40	21	21	31	29	29
A12	27	24	21	18	3	3	8	4	4
A13	2	23	10	24	43	43	20	33	43
A14	42	41	42	43	30	34	38	38	37
A15	37	43	39	42	39	42	42	43	42
A16	3	6	1	1	2	2	1	1	2

Table 4 The rank of each customer using different ranking methods (continued)

Customer	*SE*	*DR/ DEA*	*CCA/ DEA*	*GE*	*TOPSIS*	*VICOR*	*COPRAS*	*SPOTIS*	*EDAS*
A17	12	4	3	7	33	30	17	18	21
A18	10	2	2	2	23	23	7	13	18
A19	16	14	43	34	28	24	19	19	23
A20	40	38	36	36	20	22	9	15	20
A21	5	7	4	12	29	28	27	23	27
A22	39	29	41	39	24	27	23	21	25
A23	1	3	6	5	1	1	2	2	1
A24	15	9	14	8	35	29	21	20	26
A25	26	31	40	38	32	33	33	30	32
A26	41	34	37	37	19	18	29	36	31
A27	8	13	30	25	27	25	28	27	24
A28	34	27	26	26	36	37	39	39	33
A29	14	20	12	17	5	5	13	11	5
A30	25	36	11	14	38	38	37	34	35
A31	35	30	29	28	15	14	26	28	19
A32	6	37	27	31	41	39	41	40	40
A33	18	8	20	23	10	10	5	9	6
A34	9	5	7	4	14	13	6	8	10
A35	4	11	18	11	16	15	12	7	14
A36	21	16	8	10	6	8	10	11	9
A37	20	18	13	9	13	11	14	10	13
A38	22	10	15	27	18	19	24	24	16
A39	23	26	19	30	9	9	22	22	15
A40	17	15	35	20	17	16	18	16	17
A41	7	17	16	6	40	36	34	26	38
A42	29	19	9	15	12	17	15	17	12
A43	31	42	31	35	42	41	43	41	41

The scores of each customer obtained by the nine ranking methods used are presented in Table 3. The codes of five ranking methods that yield these scores (TOPSIS, VICOR, COPRAS, SPOTIS, and EDAS) are in Appendix A. The rank of each customer by each ranking method was determined by descending order of the scores. The nine ranks of each customer, one for each ranking method that was used, are presented in Table 4.

Although there are differences in the ranking, there is a positive correlation between all methods (see Table 5). On average, a high ranking in one method may also indicate a high ranking in another. One can see a high positive correlation between the TOPSIS, VICOR, COPRAS, SPOTIS, and EDAS methods. However, despite the high correlation, there are some extreme differences in the ranking for some customers and inconsistency between ranking methods regarding specific customers. For example, customer A13 is ranked second by SE and last by EDAS. This phenomenon causes ambiguity regarding the correct ranking of such customers, an issue at this paper's core.

One reason for the differences in the correlations between the ranking methods is the different weights for the criteria. All the methods based on the shortest distance use identical weights (obtained in our case by AHP), as presented in Table 2. In the methods based on DEA, the weights are determined to maximise a specified objective function, which may be at the level of a single customer or globally for all customers.

Table 5 Correlation between ranking methods

	SE	*DR*	*CCA*	*GE*	*TOPSIS*	*VIKOR*	*COPRAS*	*EDAS*
DR	0.7587							
CCA	0.6719	0.7008						
GE	0.7155	0.8138	0.8771					
TOPSIS	0.1856	0.4560	0.3006	0.3858				
VIKOR	0.2841	0.5349	0.3393	0.4619	0.9818			
COPRAS	0.5115	0.7409	0.5361	0.6465	0.7995	0.8253		
EDAS	0.5452	0.7597	0.5295	0.6972	0.8192	0.8576	0.9591	
SPOTIS	0.3834	0.6708	0.4613	0.5846	0.9345	0.9511	0.9037	0.9357

The results in Table 5 show high correlations between the methods based on the shortest distance from the ideal solution. The reasons for this are probably the use of identical weights for the criteria and a similar objective function based on distances. Nevertheless, even when the correlation is very high, there are differences in the rankings of many customers, and for some customers, the differences are high. For example, the correlation between TOPSIS and VICOR is 0.9819, and despite that, there are two customers with a difference of six steps between their ranks, one obtained by TOPSIS and one by VICOR.

Applying steps 5 and 6 of the proposed model (ranking the customers according to the average of the nine rankings and calculating the standard deviation and the confidence interval for the average of the ranking) yields the results shown in Table 6. The confidence intervals for the average of the rankings (scores) were calculated using $\alpha/2 = 2.5\%$.

We emphasise that the confidence interval refers to the average ranking score of each customer (the average ranking of the nine-ranking method) and not to the final relative ranking. That is because the final ranking is a relative integer, so even a small change in the average may have too great an effect on the ranking, and the final rank is not necessarily in the centre of the confidence interval of the average. The upper and lower values of the confidence interval are entitled up and down in Table 6.

It is possible to give each ranking method a different weight according to the correlation coefficient. The ranking method with the lowest sum of correlation coefficients (SE in this case) gets the minimum weight. In contrast, SPOTIS, with the highest sum of correlation coefficients, gets the maximum weight. According to equations (3) and (4), the relative weight of each ranking method was calculated and presented in Table 7.

Table 6 The average of the rank and the final of each customer

Customer	*Average of the rank*	*Finale rank*	*STD*	*Down*	*Up*
A1	11.44	9	5.81	7.06	15.83
A2	10.44	5	9.06	3.62	17.27
A3	39.89	42	1.90	38.46	41.32
A4	27.78	29	5.87	23.35	32.20
A5	34.11	39	3.55	31.43	36.79
A6	6.89	3	4.28	3.66	10.12
A7	32.44	35	4.16	29.31	35.58
A8	15.67	14	6.34	10.88	20.45
A9	29.11	30	5.64	24.85	33.37
A10	26.11	26	3.89	23.18	29.04
A11	29.78	31	6.02	25.24	34.31
A12	12.44	12	9.94	4.95	19.94
A13	26.78	28	14.96	15.49	38.06
A14	38.33	40	4.27	35.11	41.55
A15	41.00	43	2.12	39.40	42.60
A16	2.11	1	1.62	0.89	3.33
A17	16.11	15	10.75	8.00	24.22
A18	11.11	7	8.67	4.58	17.65
A19	24.44	23	9.29	17.44	31.45
A20	26.22	27	11.39	17.63	34.81
A21	18.00	17	10.78	9.87	26.13
A22	29.78	31	7.77	23.92	35.64
A23	2.44	2	1.88	1.03	3.86
A24	19.67	21	9.14	12.78	26.56
A25	32.78	36	4.15	29.65	35.90
A26	31.33	34	8.08	25.24	37.42
A27	23.00	22	7.42	17.41	28.59
A28	33.00	37	5.39	28.94	37.06
A29	11.33	8	5.45	7.22	15.45
A30	29.78	31	10.58	21.80	37.76
A31	24.89	25	7.22	19.45	30.33
A32	33.56	38	11.42	24.94	42.17
A33	12.11	11	6.51	7.20	17.02
A34	8.44	4	3.43	5.86	11.03
A35	12.00	10	4.42	8.67	15.33
A36	11.00	6	4.66	7.48	14.52
A37	13.44	13	3.57	10.75	16.14
A38	19.44	19	5.34	15.42	23.47

Table 6 The average of the rank and the final of each customer (continued)

Customer	*Average of the rank*	*Finale rank*	*STD*	*Down*	*Up*
A39	19.44	19	7.23	13.99	24.90
A40	19.00	18	6.16	14.35	23.65
A41	24.44	23	13.34	14.38	34.51
A42	16.11	15	5.73	11.79	20.43
A43	38.56	41	4.85	34.90	42.21

Table 7 Weight of each ranking method

	SE	*DR*	*CCA*	*GE*	*TOPSIS*	*VICOR*	*COPRAS*	*SPOTIS*	*EDAS*	*Total sum*
Sum	5.0558	6.4356	5.4167	6.1823	5.8632	6.2360	6.9225	7.1032	6.8251	56.0404
Weight	0.0902	0.1148	0.0967	0.1103	0.1046	0.1113	0.1235	0.1268	0.1218	1

6 Conclusions

This paper proposes an expert system for ranking customers according to several output and input criteria. The paper presents nine common methods for ranking customers. The main problem that this paper deals with is that each ranking method produces different ranks of the customers, making it difficult to select the best ranking method because each method has advantages and disadvantages.

We propose to use several ranking methods for all customers. Because each method provides its own ranking to each customer, all will accumulate the same number of rankings based on the same number of methods. Each customer's accumulated rankings will be averaged. The final rankings of those averages will determine each customer's position in the matrix.

We also propose a method that gives a relative weight to each ranking method based on the correlation coefficients of the ranks among all the ranking methods. We recommend choosing objective methods that are simple and easy to use, where the criteria values are taken from the CRM system. In this paper, we used two ranking methods: those related to DEA, where the weights are objectively determined, and those based on the shortest distance, where AHP determines the weights. The main advantage of the proposed expert system is that it enables us to calculate the standard deviation and the confidence intervals of the rank of each customer. These enable us to conduct a statistical examination of the reliability of the ranks and to evaluate the subjective assessments given to customers by the marketing team. The proposed method was successfully implemented in the case study. More broadly, the proposed method is applicable, with the necessary modifications, to any organisation that wants to rank its customers according to multiple input and output criteria or to select the best alternative among several alternatives.

Our study has several limitations. There are many different ranking methods from several types and families, so it is impractical to use them all. Determining how many ranking methods and which should be applied remains an open issue. This paper incorporates nine common ranking methods that are widely used – ones that are objective, intuitive, and easy to calculate. Our method can identify a high variation in the rank of a particular customer. Identifying a customer's high-rank variation indicates that the rank of such a customer obtained by the proposed method is ambiguous. In our opinion, this is also important information that should cause decision-makers to investigate the status of such customers further. Another limitation is that the weights of the criteria in some of the applied ranking methods must be determined subjectively. The solution to this problem in our paper was to determine the weights by AHP.

Acknowledgements

The authors express gratitude for the contribution of the *Int. J. of Business Forecasting and Marketing Intelligence* editor and referees to the quality of this paper for their constructive and helpful comments. Their comments and remarks truly assisted in enhancing the readability and quality of this paper.

References

Adler, N., Friedman, L. and Sinuany-Stern, Z. (2002) 'Review of ranking methods in the data envelopment analysis context', *European Journal of Operational Research*, Vol. 140, No. 2, pp.249–265.

Aires, R.F.D.F. and Ferreira, L. (2018) 'The rank reversal problem in multi-criteria decision making: a literature review', *Pesquisa Operacional*, Vol. 38, No. 1, pp.331–362.

Alaswad, F., Muruganantham, B. and Bouchi, A. (2021) 'Categorising customers and predicting their buying patterns taking into account the customer evaluation model FM', *International Journal of Business Forecasting and Marketing Intelligence*, Vol. 7, No. 2, pp.124–142.

Aldamak, A. and Zolfaghari, S. (2017) 'Review of efficiency ranking methods in data envelopment analysis', *Measurement*, Vol. 106, No. 1, pp.161–172.

Andersen, P. and Petersen, N.C. (1993) 'A procedure for ranking efficient units in data envelopment analysis', *Management Science*, Vol. 39, No. 10, pp.1261–1264.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984) 'Some models for estimating technical and scale inefficiencies in data envelopment analysis', *Management Science*, Vol. 30, No. 9, pp.1078–1092.

Behzadian, M., Otaghsara, S.K., Yazdani, M. and Ignatius, J. (2012) 'A state-of the-art survey of TOPSIS applications', *Expert Systems with Applications*, Vol. 39, No. 17, pp.13051–13069.

Benitez, J.M., Martín, J.C. and Román, C. (2007) 'Using fuzzy number for measuring quality of service in the hotel industry', *Tourism Management*, Vol. 28, No. 2, pp.544–555.

Charnes, A., Cooper, W.W. and Rhodes, E. (1978) 'Measuring the efficiency of decision making units', *European Journal of Operational Research*, Vol. 2, No. 6, pp.429–444.

Christy, A.J., Umamakeswari, A., Priyatharsini, L. and Neyaa, A. (2021) 'RFM ranking – an effective approach to customer segmentation', *Journal of King Saud University-Computer and Information Sciences*, Vol. 33, No. 10, pp.1251–1257.

de Farias Aires, R.F. and Ferreira, L. (2019) 'A new approach to avoid rank reversal cases in the TOPSIS method', *Computers & Industrial Engineering*, Vol. 132, No. 1, pp.84–97.

Dezert, J., Tchamova, A., Han, D. and Tacnet, J.M. (2020) 'The SPOTIS rank reversal free method for multi-criteria decision-making support', in *2020 IEEE 23rd International Conference on Information Fusion (FUSION)*, IEEE, July, pp.1–8.

Djurisic, V., Kascelan, L., Rogic, S. and Melovic, B. (2020) 'Bank CRM optimization using predictive classification based on the support vector machine method', *Applied Artificial Intelligence*, Vol. 34, No. 12, pp.941–955.

El Essawi, N. and El Aziz, R.A. (2012) 'Determining the main dimensions that affect e-customer relationship management readiness in the Egyptian banking industry', *International Journal of Electronic Customer Relationship Management*, Vol. 6, No. 3, pp.217–234.

Ferreira, L., Borenstein, D., Righi, M.B. and de Almeida Filho, A.T. (2018) 'A fuzzy hybrid integrated framework for portfolio optimization in private banking', *Expert Systems with Applications*, Vol. 92, No. 1, pp.350–362.

Friedman, L. and Sinuany-Stern, Z. (1997) 'Scaling units via the canonical correlation analysis in the DEA context', *European Journal of Operational Research*, Vol. 100, No. 3, pp.629–637.

Ganley, J.A. and Cubbin, J.S. (1992) *Public Sector Efficiency Measurement: Applications of Data Envelopment Analysis*, Amsterdam, North Holland.

Ghorabaee, M.K., Zavadskas, E.K., Olfat, L. and Turskis, Z. (2015) 'Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS)', *Informatica*, Vol. 26, No. 3, pp.435–451.

Gupta, S. and Lehmann, D.R. (2003) 'Customers as assets', *Journal of Interactive Marketing*, Vol. 17, No. 1, pp.9–24.

Hadad, Y. and Hanani, M.Z. (2011) 'Combining the AHP and DEA methodologies for selecting the best alternative', *International Journal of Logistics Systems and Management*, Vol. 9, No. 3, pp.251–267.

Hadad, Y. and Keren, B. (2022) 'A decision-making support system module for customer segmentation and ranking', *Expert Systems*, Vol. 40, No. 2, pp.1–13.

Hruschka, H. (1986) 'Market definition and segmentation using fuzzy clustering methods', *International Journal of Research in Marketing*, Vol. 3, No. 2, pp.117–134.

Izadikhah, M. and Farzipoor Saen, R. (2020) 'Ranking sustainable suppliers by context-dependent data envelopment analysis', *Annals of Operations Research*, Vol. 293, No. 2, pp.607–637.

Jo-Ting, W., Shih-Yen, L. and Hsin-Hung, W. (2010) 'A review of the application of RFM model', *African Journal of Business Management*, Vol. 4, No. 19, pp.4199–4206.

Karim, R. and Karmaker, C.L. (2016) 'Machine selection by AHP and TOPSIS methods', *American Journal of Industrial Engineering*, Vol. 4, No. 1, pp.7–13.

Keramati, A., Samadi, H., Nazari-Shirkouhi, S. and Askari, N. (2012) 'Identifying and prioritising critical success factors for CRM implementation: a case study', *International Journal of Electronic Customer Relationship Management*, Vol. 6, No. 3, pp.235–256.

Kim, S.Y., Jung, T.S., Suh, E.H. and Hwang, H.S. (2006) 'Customer segmentation and strategy development based on customer lifetime value: a case study', *Expert Systems with Applications*, Vol. 31, No. 1, pp.101–107.

Lambert, D.M. and Enz, M.G. (2017) 'Issues in supply chain management: progress and potential', *Industrial Marketing Management*, Vol. 62, No. 1, pp.1–16.

Lichtenstein, S. and Slovic, P. (1971) 'Reversals of preference between bids and choices in gambling decisions', *Journal of Experimental Psychology*, Vol. 89, No. 1, pp.46–67.

Lichtenstein, S. and Slovic, P. (1973) 'Response-induced reversals of preference in gambling: an extended replication in Las Vegas', *Journal of Experimental Psychology*, Vol. 101, No. 1, pp.16–23.

Mi, X., Tang, M., Liao, H., Shen, W. and Lev, B. (2019) 'The state-of-the-art survey on integrations and applications of the best worst method in decision making: why, what, what for and what's next?', *Omega*, Vol. 87, No. 1, pp.205–225.

Nazari, A., Mehregan, M. and Tehrani, R. (2020) 'Evaluating the effectiveness of data mining techniques in credit scoring of bank customers using mathematical models: a case study of individual borrowers of Refah Kargaran bank in Zanjan Province, Iran', *International Journal of Nonlinear Analysis and Applications*, Vol. 11, No. 1, pp.299–309.

Nie, D., Cappellari, P. and Roantree, M. (2021) 'A methodology for classification and validation of customer datasets', *Journal of Business & Industrial Marketing*, Vol. 36, No. 5, pp.821–833.

Nikumanesh, E. and Albadvi, A. (2014) 'Customer's life-time value using the RFM model in the banking industry: a case study', *International Journal of Electronic Customer Relationship Management*, Vol. 8, No. 1, pp.2–13.

Noorizadeh, A., Mahdiloo, M. and Saen, R.F. (2013) 'Evaluating relative value of customers via data envelopment analysis', *Journal of Business & Industrial Marketing*, Vol. 28, No. 7, pp.577–588.

Opricovic, S. (1998) *Multicriteria Optimization of Civil Engineering Systems*, Faculty of Civil Engineering, Belgrade, Vol. 2, No. 1, pp.5–21.

Opricovic, S. and Tzeng, G.H. (2004) 'Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS', *European Journal of Operational Research*, Vol. 156, No. 2, pp.445–455.

Opricovic, S. and Tzeng, G.H. (2007) 'Extended VIKOR method in comparison with outranking methods', *European Journal of Operational Research*, Vol. 178, No. 2, pp.514–529.

Pamučar, D., Stević, Ž. and Sremac, S. (2018) 'A new model for determining weight coefficients of criteria in MCDM models: full consistency method (FUCOM)', *Symmetry*, Vol. 10, No. 9, pp.1–22.

Rezaei, J. (2016) 'Best-worst multi-criteria decision-making method: some properties and a linear model', *Omega*, Vol. 64, No. 1, pp.126–130.

Rogić, S. and Kašćelan, L. (2021) 'Class balancing in customer segments classification using support vector machine rule extraction and ensemble learning', *Computer Science and Information Systems*, Vol. 18, No. 3, pp.893–925.

Saaty, T.L. (1980) *The Analytic Hierarchy Process*, McGraw-Hill, New York.

Sajjadi, K., Khatami-Firuzabadi, M.A., Amiri, M. and Sadaghiani, J.S. (2015) 'A developing model for clustering and ranking bank customers', *International Journal of Electronic Customer Relationship Management*, Vol. 9, No. 1, pp.73–86.

Singaravel, B. and Selvaraj, T. (2015) 'Optimization of machining parameters in turning operation using combined TOPSIS and AHP method', *Tehnički Vjesnik*, Vol. 22, No. 6, pp.1475–1480.

Singh, N., Singh, P., Singh, K.K. and Singh, A. (2020) 'Machine learning based classification and segmentation techniques for CRM: a customer analytics', *International Journal of Business Forecasting and Marketing Intelligence*, Vol. 6, No. 2, pp.99–117.

Sinuany-Stern, Z. and Friedman, L. (1998) 'DEA and the discriminant analysis of ratios for ranking units', *European Journal of Operational Research*, Vol. 111, No. 3, pp.470–478.

Sinuany-Stern, Z., Mehrez, A. and Hadad, Y. (2000) 'An AHP/DEA methodology for ranking decision making units', *International Transactions in Operational Research*, Vol. 7, No. 2, pp.109–124.

Tatsuoka, M.M. and Lohnes, P.R. (1988) *Multivariate Analysis: Techniques for Educational and Psychological Research*, Macmillan Publishing Co, Inc., Washington.

Velasquez, M. and Hester, P.T. (2013) 'An analysis of multi-criteria decision making methods', *International Journal of Operations Research*, Vol. 10, No. 2, pp.56–66.

Vogt, C.A. (2011) 'Customer relationship management in tourism: management needs and research applications', *Journal of Travel Research*, Vol. 50, No. 4, pp.356–364.

Yoon, K. and Hwang, C.L. (1985) 'Manufacturing plant location analysis by multiple attribute decision making: part I – single-plant strategy', *International Journal of Production Research*, Vol. 23, No. 2, pp.345–359.

Zavadskas, E.K., Govindan, K., Antucheviciene, J. and Turskis, Z. (2016) 'Hybrid multiple criteria decision-making methods: a review of applications for sustainability issues', *Economic Research-Ekonomska Istraživanja*, Vol. 29, No. 1, pp.857–887.

Zavadskas, E.K., Kaklauskas, A. and Sarka, V. (1994) 'The new method of multicriteria complex proportional assessment of projects', *Technological and Economic Development of Economy*, Vol. 1, No. 3, pp.131–139.

Appendix A

Codes

The experiment results for algorithms TOPSIS, VICOR, COPRAS, SPOTIS and EDAS, with AHP, were obtained by the following code implemented in IPython platform infrastructure and pandas, np, and pymcdm open-source Python libraries.

```
# Import libraries
import pandas as pd
import numpy as np
from pymcdm import weights as mcdm_weights
from pymcdm import methods as mcdm_methods
from pymcdm import methods as mcdm_methods
from pymcdm import weights as mcdm_weights
from pymcdm.helpers import rankdata
```

```
# Import data
df = pd.read_excel (r'data.xlsx', sheet_name='data')
matrix = df[df.columns[1:]].to_numpy()
types = np.array([–1, –1, –1, –1, –1, –1, 1, 1, 1, 1])
```

```
# Weights shemas
y_ahp = [0.073937, 0.026164, 0.073937, 0.303804, 0.218355,
          0.303804, 0.248052, 0.089613, 0.042897, 0.619438]
ahp = [float(i)/sum(y_ahp) for i in y_ahp]
weight_schemas = {'EQUAL' : mcdm_weights.equal_weights(matrix),
                  'AHP': np.array(ahp),}
```

```
method = 'TOPSIS'
rs_temp = df.copy(deep = True)
rank_methods = { 'TOPSIS': mcdm_methods.TOPSIS(),}
for rn, rank_method in rank_methods.items():
        for wn, weight_schema in weight_schemas.items():
        t = rank_method(matrix, weight_schema, types)
        r = rankdata(t, reverse = True)
        rs_temp[rn + '_' + wn] = t
        rs_temp[rn + '_' + wn + '_RANK'] = r
```

```
method = 'VIKOR'
rank_methods = {'VIKOR': mcdm_methods.VIKOR(),}
for rn, rank_method in rank_methods.items():
```

```
    for wn, weight_schema in weight_schemas.items():
        t = rank_method(matrix, weight_schema, –1*types)
        r = rankdata(t, reverse = True)
        rs_temp[rn + '_' + wn] = t
        rs_temp[rn + '_' + wn + '_RANK'] = r
```

```
method = 'COPRAS'
rank_methods = {'COPRAS': mcdm_methods.COPRAS(),}
for rn, rank_method in rank_methods.items():
    for wn, weight_schema in weight_schemas.items():
        t = rank_method(matrix, weight_schema, types)
        r = rankdata(t, reverse = True)
        rs_temp[rn + '_' + wn] = t
        rs_temp[rn + '_' + wn + '_RANK'] = r
```

```
method = 'EDAS'
rank_methods = {'EDAS': mcdm_methods.EDAS(),}
for rn, rank_method in rank_methods.items():
    for wn, weight_schema in weight_schemas.items():
        t = rank_method(matrix, weight_schema, types)
        r = rankdata(t, reverse = True)
        rs_temp[rn + '_' + wn] = t
        rs_temp[rn + '_' + wn + '_RANK'] = r
```

```
method = 'SPOTIS'
rank_methods = { 'SPOTIS': mcdm_methods.SPOTIS(),}
bounds = np.vstack((
    np.min(matrix, axis=0),
    np.max(matrix, axis=0)
)).T
for rn, rank_method in rank_methods.items():
    for wn, weight_schema in weight_schemas.items():
        t = rank_method(matrix, weight_schema, –1*types, bounds)
        r = rankdata(t, reverse = True)
        rs_temp[rn + '_' + wn] = t
        rs_temp[rn + '_' + wn + '_RANK'] = r
```

Appendix B

Table B1 The selected criteria

Sign	Service cost criteria (inputs)	Sign	Customer value criteria (outputs)
X_1	Returns of sold products (NIS)	Y_1	Sales
X_2	Number of orders	Y_2	Pays in line with payment terms (percent)
X_3	Distance from plants (km)	Y_3	Loyalty – time from the first-ever purchase (months)
X_4	Cost of transportation (NIS)	Y_4	Profitability rate (percent)
X_5	Number of complaints		
X_6	Average days from supply to payment		

Table B2 The values of the inputs and outputs of each customer (matrix P)

Customer	Inputs						Outputs			
	X_1	X_2	X_3	X_4	X_5	X_6	Y_1	Y_2	Y_3	Y_4
A1	955.78	2	46.36	396.21	1	68	50,580	97%	68	34%
A2	115.08	1	90.46	251.93	2	13	7,463	100%	24	36%
A3	1,287.05	7	62.50	1,010.51	6	55	47,542	53%	15	21%
A4	1,284.31	5	64.06	798.65	2	32	27,829	91%	82	17%
A5	527.60	8	79.33	1,110.17	4	63	52,855	25%	90	23%
A6	705.27	2	35.35	505.67	1	33	65,307	96%	43	27%
A7	1,571.93	8	86.10	1,208.95	6	17	50,743	74%	11	23%
A8	1,376.46	2	43.74	463.24	2	62	38,807	100%	83	34%
A9	667.74	4	41.76	750.38	6	47	27,373	94%	69	28%
A10	587.24	2	51.47	360.09	2	50	13,618	100%	73	22%
A11	535.85	7	4.94	1,018.15	5	28	37,120	34%	14	26%
A12	1,429.20	1	76.57	377.05	2	22	29,459	100%	5	37%
A13	5,693.95	17	91.63	4,397.85	3	77	232,196	55%	44	12%
A14	1,972.28	4	35.55	665.56	3	88	50,747	32%	7	24%
A15	205.22	7	78.03	1,064.41	6	76	46,332	34%	51	20%
A16	100.68	2	48.25	307.24	1	4	21,550	64%	30	37%
A17	1,083.76	4	15.12	786.29	1	18	54,389	80%	66	14%
A18	219.40	4	6.64	625.35	2	10	46,160	47%	54	19%
A19	409.70	1	34.87	192.97	1	54	10,463	100%	19	20%
A20	191.55	2	40.14	325.95	1	30	12,426	30%	32	23%
A21	514.47	11	17.66	1,475.56	4	7	68,848	75%	61	18%
A22	350.21	2	28.99	328.64	1	65	22,072	53%	29	23%
A23	943.31	1	1.68	306.12	2	11	23,670	100%	8	40%
A24	1,929.19	1	27.85	375.69	1	48	56,971	100%	46	15%
A25	486.45	2	92.52	580.37	1	85	22,747	100%	62	21%

Table B2 The values of the inputs and outputs of each customer (matrix P) (continued)

Customer	*Inputs*						*Outputs*			
	X_1	X_2	X_3	X_4	X_5	X_6	Y_1	Y_2	Y_3	Y_4
A26	1,934.99	10	96.39	1,518.75	4	67	55,024	100%	47	30%
A27	1,276.38	7	8.80	1,046.33	1	87	53,432	100%	83	24%
A28	2,190.40	9	26.14	1,171.44	4	36	45,205	100%	41	18%
A29	1,197.47	6	92.33	914.92	1	33	39,565	100%	78	36%
A30	2,481.14	6	55.75	954.33	5	30	60,699	56%	82	17%
A31	823.06	6	83.53	831.17	4	38	33,771	97%	51	30%
A32	103.92	5	29.75	812.75	4	53	37,274	30%	84	13%
A33	330.05	2	29.34	550.26	1	20	10,000	97%	31	28%
A34	342.36	1	27.10	306.58	2	31	31,266	100%	80	27%
A35	176.08	1	87.26	321.93	1	64	55,681	100%	80	28%
A36	681.24	2	58.39	503.96	3	30	47,629	80%	74	33%
A37	314.13	1	53.85	247.33	5	15	27,399	100%	40	29%
A38	999.86	9	61.74	1,441.45	1	21	51,058	100%	18	25%
A39	1,957.43	12	76.57	1,619.55	1	44	67,235	73%	50	32%
A40	635.24	1	15.48	185.86	3	65	30,341	100%	48	29%
A41	2,458.84	1	27.51	227.22	5	38	73,282	100%	16	8%
A42	1,312.36	5	49.30	845.20	1	24	50,487	50%	46	27%
A43	439.94	6	64.37	1,004.17	2	55	32,334	28%	86	9%
Average	1,041.8	4.6	49.9	795.0	2.7	42.2	44,626.8	77%	49.33	24%