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American financial markets dependencies: a vine copula approach

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Abstract: Regular vine copulas are used to evaluate the dependence between American financial markets (Argentina, Brazil, Canada, Chile, Colombia, Mexico, Peru, and the USA) from August 16, 2011, to April 21, 2022. The behaviour of marginal distributions is described by AR(1)-TGARCH models with errors distributed as an asymmetric skewed Student's t, which are adequate to model returns and their volatility. The conditional dependency between pairwise countries is estimated for the covid period, and three subperiods are analysed, pre-covid, covid, and post-covid. It is found that the contagion routes between the different American countries have the USA as the root node.

Keywords: vine copulas; TGARCH; dependence.

JEL codes: C22, C32, C51, E44.

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Biographical notes: Arturo Lorenzo-Valdes is a Doctor of Administration specialising in Finance from the Tecnológico de Monterrey, Campus Ciudad de México. He has taught different courses in financial econometrics, financial mathematics, actuarial mathematics, and financial theory for more than 18 years at Bachelor, Master, and Doctoral levels in different higher education institutions. He has attended conferences, published articles, and participated in research projects on empirical finance issues, mainly in non-linear time series. He is a Mexican National System of Researchers (SNI) Level 2 member.

1 Introduction

Portfolio diversification is a critical factor for investors. When markets are highly interdependent, it is challenging to diversify investments. This dependence on financial markets facilitates contagion in times of crisis, whether the origin is financial, in some other sector of the economy, due to natural disasters or pandemics such as covid. This prompts us to study the connection between financial markets, apply copula methods to measure this relationship that could lead to contagion, and determine what type of dependency exists.

Many analysis proposals have used statistical and econometric tools to describe the dependency and joint movements between economic and financial variables. The most widely used measure is Pearson's correlation, which calculates the linear dependence between two quantitative variables. The drawback of this method is that if the distribution of the data does not present a linear behaviour, the dependence between the variables is not adequately explained; for example, the linear correlation does not explain the non-linear dependence or the dependence of the tails of the variables (that would measure the relationship between extreme returns). One of the tools that are becoming very popular in its application is the copula method since it allows us to understand the dependency at a deeper level.

Using copulas, instead of a simple correlation study, allows us to describe the dependence between random variables and their respective MDF separately. In addition, copulas allow describing the dependence of extreme changes in financial returns (Chollete et al., 2008), known as tail dependence. Another benefit is that adapting the Monte Carlo processes to perform the simulations the technique requires can be carried out with relative ease.

Authors such as Costinot et al. (2000) point out that the dependency structure, particularly in periods of crisis and between financial markets, is better described through tools such as copulas instead of simple correlation analysis. Considering the extreme dependency observed between international stock markets, copula methods are much more effective in explaining crisis events.

As the copulas allow us to describe the joint behaviour of the financial time series, we can model the relationships observed in international markets and observe, as Longin and Solnik (1995) mention, that the dependence is more potent during periods of high volatility.

Therefore, the individual behaviour of stock market index returns and their volatility can be described with models from the ARCH family, such as TGARCH and EGARCH.

A method that allows calculating a multivariate distribution as a product of the combinations of bivariate copulas is proposed by Joe (1997), known as pairwise copulas, through the construction of vine copulas. These methods use flexible structures to model dependency using graph theory. The shapes of the different graphs provide the maximum pairwise dependence of the financial variables. Among these structures are the C-Vine and D-Vine models, exceptional cases of regular Vine copulas.

This article uses vine copulas techniques to find the contagion structure among the eight American markets.

The rest of this document is divided as follows: the second section presents a brief review of the literature on copulas in financial markets; in the next section, the concept of copula and its dependency measures are discussed. Subsequently, the vine copulas are discussed to describe the structure between several financial variables through a graph that relates them in a bivariate way. The fifth part explains the behaviour of the marginals as GARCH-type models; the data and results are shown in the sixth section to finish with the conclusions in this regard finally.

2 Literature review

Among the authors who have carried out studies on the relationship between Latin American stock indices is Choudry (1997), who finds long-term relationships between the stock indices of six Latin American countries through error correction models (ECM).

Christofi and Pericli (1999) estimate an autoregressive vector (VAR) with volatilities estimated with GARCH-type models between five Latin American stock markets and find linear and quadratic dependence between their returns.

Arouri et al. (2010) used GARCH models to estimate dynamic correlations in different periods. They found that the relationship has increased significantly since 1994 and in times of crisis.

Among the authors who used the copula analysis methodology are Canela and Pedreira (2012), Rodríguez (2007), and Okimoto (2008), who carry out similar works to this one but do not include the crisis that arose in 2008.

The former applied two-dimensional copulas to study the pairwise dependence structures of the daily returns of Argentina, Brazil, Chile, Mexico, Peru, and Venezuela. They concluded that the dependency between stock markets in Latin America presents a greater probability of extreme losses, so we can expect that the structure of dependency between them will be stronger in periods of crisis than in those of calm and stability.

Rodríguez (2007) used the daily returns on the stock indices of Thailand, Malaysia, Indonesia, Korea, and the Philippines, during the Asian crisis and Mexico, Argentina, Brazil, and Chile during the Mexican crisis.

Okimoto (2008) focused on stock market indices in the USA and the UK. The last two studies used regime change copulas to find evidence of changes in the dependency structure during periods of crisis.

These studies concluded that dependencies between stock markets present a higher probability of extreme losses. Therefore, we expect that the dependency structure will be strengthened in periods of crisis. Lorenzo-Valdés and Massa (2013) studied the dependency between Mexico and Brazil with three different types of copulas and in different periods, also finding an increase in dependency in periods of crisis.

Bucio et al. (2016) identify how the patterns of potential losses vary between the stock markets of Mexico and the USA by measuring the value at risk (VaR) through elliptic copulas.

Ortiz et al. (2016) applied copula models to estimate the degree of dependency between seven Latin American markets, Canada and the USA, and evaluate them through VaR. The authors suggest that the copula theory leads to more realistic estimates than traditional methodologies in the estimation of the degrees of dependency, as well as in the estimation of the VaR.

Behaviour in the top and bottom tails may differ. This generally happens in periods of financial crises and can be studied in copula analysis by measuring changes in the dependence of the tail distribution, for which the copula must vary over time.

Patton (2001) introduced the concept of a conditional copula that allows analysis to be carried out by including the conditional (time-dependent) density for each variable and the conditional dependency between them. In his work, he studies the asymmetries in the dependency structure of the German mark and the Japanese yen concerning the US dollar. In subsequent works, Patton (2006a) developed a bivariate model using the symmetric Joe-Clayton copula with a GARCH model to describe the evolution of the conditional variances of returns. This model allowed the copula parameters to be functions of time and to describe the dependency between them during extreme events.

Lorenzo-Valdés (2016) follows Patton and studies the conditional dependence of the Mexican and US stock markets. He uses AR-TGARCH and AR-EGARCH models for the marginal distributions and finds high dependence on tails in periods of high volatility.

Similarly, Johansson (2011) applied conditional dependency to the financial markets of Europe and East Asia in the context of the 2008 crisis and concluded that regional volatility and joint movements were higher during the 2008 crisis than during the Asian financial crisis of the 1990s. More recently, Czapkiewicz and Majdosz (2014) used copulas to find the dynamic interdependence and similarities between European, American, and Asian markets.

Instead of focusing on special copulas between countries, the authors analyse the regional dependency of the tails. Their main finding is that regional volatility and joint movements in Europe and East Asia were more significant during the 2008 global financial crisis than during the entire Asian financial crisis. However, the effect on Europe is more significant.

Santillán et al. (2018) obtain measures of the level of bivariate dependency in tails between the Mexican stock index and three Latin American stock indices.

Regarding vine copulas, Loaiza et al. (2015) implement a methodology of regular vine copulas to assess the level of contagion between the exchange rates of six Latin American countries in terms of tail dependency coefficients.

Also, for the exchange rate, Yuan and Tang (2018) use the Vine copula model to analyse the impact of exchange rate volatility on exports.

Allen et al. (2017) present the use of regular vine copulas in an analysis of the codependencies of ten European stock markets and find that the dependencies change in a complex way and are subject to change in different economic circumstances.

Stübinger et al. (2018) developed a multivariate arbitrage strategy in the USA market and modelled the dependency of the upper and lower tails varying over time.

Gurgul and Machno (2016) studied the dependency structure between 12 European markets and compared it with 12 Asia-Pacific markets. They employed a two-state regime switch model and dependency via a regular vine copula. In their study, they confirm the existence of asymmetric dependencies and fat tails and that the European markets have proven to be more strongly connected than the Asia-Pacific markets.

Arreola et al. (2017) use regular vine copulas to examine risk through dependency measures of three different portfolios. They conclude that dependency measures can be used to develop risk, investment, and hedging strategies that give more adequate results.

Finally, Lorenzo-Valdes (2020) applies a methodology of regular vine copulas to evaluate the level of dependence between the financial markets of six Latin American countries and find that Latin American stock markets have a probability of extreme losses greater than a probability of extreme profits and that the dependence measure increases in crisis periods.

3 Copulas and dependency measures

The dependency structure between the variables can be modelled by employing a mechanism that expresses the cumulative distribution function (CDF) from the marginal distribution functions (MDF). A copula $C(u_1, u_2, \ldots, u_n)$ is an CDF for n uniform variables over the unit interval. Sklar's theorem (1959) says that if we take $u_j = F_j(x_j)$ for $j = 1, \ldots n$ to be the CDF of a univariate continuous random variable X_j, then $C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$ is a multivariate distribution function for $X = (X_1, X_2, \ldots, X_n)$ with marginal distributions F_j, $j = 1, \ldots n$. Conversely, if F is a multivariate continuous CDF with univariate marginals F_j, $j = 1, \ldots n$, then a unique multivariate copula C exists such that $F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$.

The properties of copulas allow us to study dependencies more easily in financial markets. Among these properties, we first mention that copulas are invariant to monotone transformations of random variables. Second, there is a direct relationship between the parameters of copulas and measures of concordance, as widely used by Kendall's tau. Third, they provide an asymptotic dependence treatment in the tails of the distributions.

Kendall's tau (τ) is a measure of concordance between two random variables. Two points (x_1, x_2) and (y_1, y_2) are said to be concordant if $(x_1–y_1)(x_2–y_2) > 0$, and discordant if $(x_1–y_1)(x_2–y_2) < 0$. Likewise, two random vectors (X_1, X_2), and (Y_1, Y_2) are concordant if the probability $P[(X_1–Y_1)(X_2–Y_2) > 0]$ is greater than $P[(X_1–Y_1)(X_2–Y_2) < 0]$; that is, X_1 and X_2 tend to increase together. They are discordant if the opposite happens. Kendall's τ measures differences in probability:

$$\tau(X_1, X_2) = P\left[(X_1, X_1)(X_2 - Y_2) > 0\right] - P\left[(X_1 - Y_1)(X_2 - Y_2) < 0\right]. \tag{1}$$

Kendall's τ is related to copulas through the following equation:

$$\tau(X_1, X_2) = 4\iint C(u_1, u_2)\, dC(u_1, u_2) - 1. \tag{2}$$

Another dependence measure defined by copulas is the asymptotic tail dependence, which measures the performance of random variables during extreme events. In this paper, we use a parameter to measure the probability that a drastic increase (decrease) in financial market returns occurs if we observe an extreme increase in another market returns.

The lower τ^L and upper τ^U asymptotic tail dependence coefficients are defined as:

$$\begin{aligned} \tau^L &= \lim_{\alpha \to 0^+} P\left(X_2 < F_2^{-1}(\alpha) \mid X_1 < F_1^{-1}(\alpha)\right) = \lim_{\alpha \to 0^+} \frac{C(\alpha, \alpha)}{\alpha}, \\ \tau^U &= \lim_{\alpha \to 1^-} P\left(X_2 > F_2^{-1}(\alpha) \mid X_1 > F_1^{-1}(\alpha)\right) = \lim_{\alpha \to 1^-} \frac{1 - 2\alpha + C(\alpha, \alpha)}{1 - \alpha}\alpha \end{aligned} . \tag{3}$$

There is independence in the tail if the value is zero and dependence if the value is between zero and one. If the value is one, there is perfect dependence.

For this study, the conditional tail dependence measures were estimated following Patton (2006b). The lower τ^L and upper τ^U tail coefficients are supposed to be time-dependent. The evolving dynamic is as follows:

$$\begin{aligned} \tau^L &= \Lambda\left(\lambda_{0L} + \lambda_{1L}\tau_{t-1}^L + \lambda_{2L}\left|u_{1t-1} - u_{2t-1}\right|\right), \\ \tau^U &= \Lambda\left(\lambda_{0U} + \lambda_{1U}\tau_{t-1}^U + \lambda_{2U}\left|u_{1t-1} - u_{2t-1}\right|\right) \end{aligned} \tag{4}$$

where Λ is the logistic function that keeps the values between zero and one.[1]

4 Copulas vine

Vine copulas are a flexible way of describing multivariate copulas using bivariate copulas.

This technique, initially developed by Joe (1997), consists of decomposing the n-dimensional density into bivariate pairs. In the vine copulas, there are $\frac{n(n-1)}{2}$ pairs of copulas arranged in a tree with n nodes, which allows us to formulate several bivariate copulas considering several financial markets.

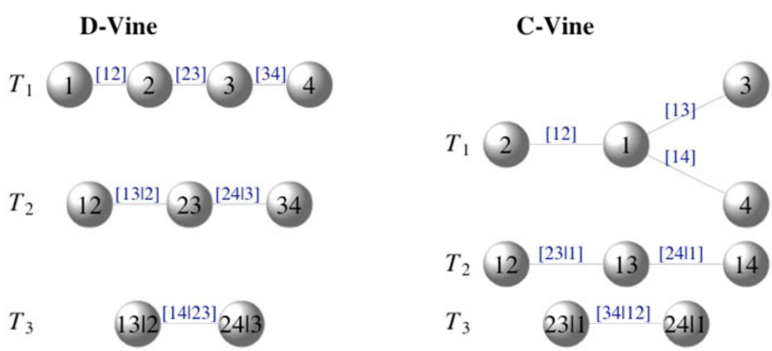
According to Bedford and Cook (2001), V is a regular vine copula of size n with $E(V)=\bigcup_{i=1}^{n} E_i$ denoting the set of edges in V if:

1 consists of $n-1$ trees, that is, $V = \{T_1, ..., T_{n-1}\}$

2 the tree T_1 is connected with nodes $N_1 = \{1,\dots, n-1\}$ and edges E_i and for $j = 2, \dots n-1$, $N_j = E_{j-1}$ (the edges of tree j become nodes of tree $j + 1$)

3 (proximity) for $j = 2, \dots n-1$, the edge $\{a, b\} \in E_j$ with $\#(a\Delta b) = 2$ where # denotes cardinality and Δ symmetric difference (only one edge joins two nodes).

Figure 1 Example of D-Vine and C-Vine for four markets (see online version for colours)



There are different structures of regular vine copulas in which the tree's shape is not imposed, nor is the copula used on each edge, and each model specifies how to decompose the distribution. There are two canonical classes of vine copulas presented by Aas et al. (2009), namely, the D-Vine copulas in which the cardinality of every node cannot be greater than two and the canonical vine copulas (C-Vine) in which each tree T_j has a single node of degree $n-j$. To exemplify, we want to explain the dependency between the financial markets of four countries. Figure 1 shows a D-Vine and a C-Vine with four variables. In the case of D-Vine, the first tree describes the dependency through bivariate copulas between market 1 and 2, 2 with 3, and 3 with 4; the second tree describes the dependency through copulas between market 1 and 3 but conditioned to the behaviour of market 2 [13|2] and the dependency between market 2 and market 4 conditioned to the behaviour of market 3. Finally, the third tree measures the dependence between markets 1 and 4 conditioned to the behaviour of markets 2 and 3. It is assumed that we will have bivariate copulas for each edge. A root node is selected in each tree for a C-Vine, and all pairwise dependencies for this node are modelled conditional on all previous root nodes–each structure, whether D-Vine or C-Vine, is assembled by selecting a specific order of the variables.

Then, in the first tree, the dependency of the first and second variables of the second and third of the third and fourth, and so on, is modelled using bivariate copulas.

Having used pairwise copula constructions, the density of the copula for the n variables can be expressed as a product of the conditional copulas.

The principle of Brechmann and Schepsmeier (2013) is followed to select the order of variables in the trees. This principle is based on a sequential selection of trees with empirical optimisation rules. The rule for selecting the first tree indicates an order of the variables that aims to capture the most significant possible dependency. The root node is chosen when the sum of the absolute values of Kendall's Tau of all pairs is maximised. The idea is that the first tree should contain the bivariate copulas with the strongest dependency. It is necessary to consider that the copulas in each edge can be different. Therefore, two steps must be followed:

1 estimate the parameters of the copula

2 validate the choice of the copula.

This last process consists of performing a goodness-of-fit test based on the maximisation of Kendall's Tau, followed by using information criteria such as Akaike's (AIC) to choose the D-Vine or C-Vine copula.

5 Behaviour of marginal distributions

To describe the marginal behaviour, we use the closing financial market index for the country i at time t, P_{it}. Continuous returns are calculated as follows:

$$r_{it} = \ln P_{it} - \ln P_{it-1} \tag{5}$$

We also examined the volatility of the returns using a TGARCH model. For the conditional mean, we used an AR(1)-TGARCH (1,1) model:

$$\begin{aligned} r_{it} &= \phi_0 + \phi_1 r_{it-1} + u_{it} \\ u_{it} &= \sigma_{it}\varepsilon_{it} \\ \sigma_{it}^2 &= \alpha_0 + \alpha_1 u_{it-1}^2 + \gamma u_{it-1}^2 I\left(u_{it-1} < 0\right) + \beta\sigma_{it-1}^2 \end{aligned} \tag{6}$$

The standardised errors ε_{it} are distributed as an asymmetric standardised student's t, and the degrees of freedom (υ) are estimated.

Model (6) presents an equation for the returns that, in this case, is defined as an autoregressive process of order one as the returns for the period depend on the same returns in the previous period and an equation for the variance that serves to describe the volatility of continuous returns (in logarithms). The conditional variances are estimated with time-series models of the ARCH/GARCH family.[2]

In our case, a TGARCH model is used. This model is an extension, proposed by Zakoian (1994), of the traditional GARCH one.[3] We use it because the TGARCH model has been recognised as the best to describe asset returns volatility. The TGARCH model can mainly capture features that characterise many financial and economic series. Among them are the existence of non-constant volatilities, volatility clustering, skewed and leptokurtic distributions, and leverage effects.

From a modelling perspective, the main feature of the TGARCH model is that it allows the volatility of the return series on period t, r_t, to depend on the 'news' arriving to the market (i.e., the lagged innovation u_{t-1}).

6 Data and results

The study data are based on the financial market's daily closing price indexes of eight American countries (Argentina, Brazil, Canada, Chile, Colombia, Mexico, Peru, and USA) collected from August 16, 2011, to April 21, 2022.

Daily continuous returns are calculated as in (5), and descriptive statistics are shown in Table 1.

Table 1 Descriptive statistics of continuous returns for the eight American countries

	Argentina	*Brazil*	*Canada*	*Chile*
Mean	0.0012	0.0012	0.0012	0.0012
Median	0.0016	0.0016	0.0016	0.0016
Maximum	0.0977	0.0977	0.0977	0.0977
Minimum	–0.4769	–0.4769	–0.4769	–0.4769
Std. dev.	0.0237	0.0237	0.0237	0.0237
Skewness	–3.2985	–3.2985	–3.2985	–3.2985
Kurtosis	65.3313	65.3313	65.3313	65.3313
Jarque-Bera	452,978.3	452,978.3	452,978.3	452,978.3
p-value	0.0000	0.0000	0.0000	0.0000
	Colombia	*Mexico*	*Peru*	*SP500*
Mean	0.0012	0.0012	0.0012	0.0012
Median	0.0016	0.0016	0.0016	0.0016
Maximum	0.0977	0.0977	0.0977	0.0977
Minimum	–0.4769	–0.4769	–0.4769	–0.4769
Std. dev.	0.0237	0.0237	0.0237	0.0237
Skewness	–3.2985	–3.2985	–3.2985	–3.2985
Kurtosis	65.3313	65.3313	65.3313	65.3313
Jarque-Bera	452,978.3	452,978.3	452,978.3	452,978.3
p-value	0.0000	0.0000	0.0000	0.0000

It is observed that in all cases, the Jarque-Bera test rejects that the continuous returns have a normal distribution.

Empirical correlation and Kendall's tau are calculated and presented in Table 2.

Correlations between the different countries are shown at the bottom of the diagonal, and Kendall's Tau is at the top of the diagonal.

The correlation and Kendall's Tau take positive values, so there is a positive relationship between the returns of the countries' stock indices.

Table 2 Empirical Kendalls Tau (upper diagonal) and correlations (lower diagonal)

	Argentina	*Brazil*	*Canada*	*Chile*	*Colombia*	*Mexico*	*Peru*	*SP500*
Argentina		0.2997	0.2693	0.2096	0.1735	0.2291	0.2251	0.2536
Brazil	0.4214		0.3274	0.2754	0.2248	0.3135	0.2547	0.3242
Canada	0.3874	0.6072		0.2601	0.2366	0.3348	0.3139	0.5308
Chile	0.3090	0.4299	0.4751		0.2349	0.2978	0.2189	0.2468
Colombia	0.3000	0.4395	0.5032	0.4436		0.2250	0.2156	0.2203
Mexico	0.3344	0.5108	0.5594	0.4263	0.4006		0.2444	0.3583
Peru	0.3034	0.4309	0.5213	0.3754	0.3811	0.4033		0.2567
SP500	0.3657	0.5812	0.8057	0.4293	0.4605	0.5646	0.4430	

In the case of correlation (linear dependence), when the performance of the stock index of one country goes up (down), that of the other country does too. In the case of Kendall's Tau, there is a concordance between the variables.

The maximum linear relationship, measured by the correlation, is found between the markets of the USA (SP500) with Canada, followed by Brazil with Canada, Brazil with the USA, and the USA with Mexico.

Regarding the measure of correspondence, which indicates that 'large' values in the returns of one market tend to be associated with 'large' values in those of the other financial market and 'small' values of a market are associated with 'small' values of the other market, it is found that the most significant dependency is again between the markets of USA (SP500) with Canada followed by the USA with Mexico.

Table 3 shows the results of the estimation of the behaviour of the marginal returns assuming an AR(1)-TGARCH(1, 1) model as in (6). The estimation of the different parameters is shown, as well as their p-value.

Four panels are presented for each country. The first panel shows the estimates of the mean (of returns) equation for the AR(1) model. It can be seen that the constant ϕ_0 is significantly positive for Argentina, Canada, and the USA and statistically zero for the other ones.

In the second panel is the equation for the conditional variance. The parameters β mainly α_1 measure the effect of clusters, that is, the grouping of volatility over time, and they are significant for all countries except α_1 for Canada, Chile, and the USA.

The parameter γ measures the leverage effect that, in all cases, it is significantly positive; this indicates that when returns decrease, volatility increases.

The third panel presents the estimator of the degrees of freedom of the asymmetric t-standardised distribution and the parameter that measures asymmetry. The degrees of freedom are significant and not extensive so that we can rule out the normal distribution of the disturbances.

In the case of the parameter that measures the asymmetry of the distribution, it is found that it cannot be rejected that the value of the parameter is equal to one, which indicates that the stock indices, as assumed in the stylised facts, have a symmetric distribution, unlike other assets.

The adequacy of the fitted models is carried out by reviewing the correlograms of the standardised residuals and the squared standardised residuals whose behaviour must be white noise, which is valid in all cases.

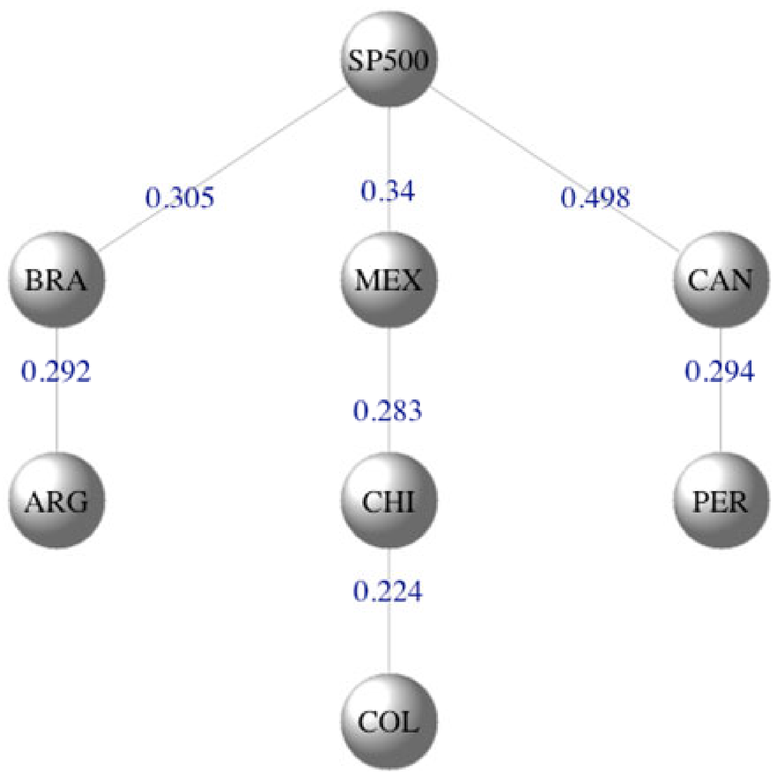
Table 3 Estimate parameters and p-value for the AR(1)-TGARCH model with standardised symmetric t distribution

	Argentina		Brazil		Canada		Chile		Colombia		Mexico		Peru		SP500	
	Coef.	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*	*Coef.*	*p-value*
ϕ_0	0.0014	0.0005	0.0003	0.1449	0.0002	0.0624	0.0000	0.9463	0.0000	0.9574	0.0000	0.8600	0.0001	0.5090	0.0004	0.0003
ϕ_1	0.1033	0.0000	–0.0215	0.2720	0.0224	0.2597	0.1513	0.0000	0.1167	0.0000	0.0534	0.0057	0.1006	0.0000	–0.0444	0.0217
α_0	0.0000	0.0001	0.0000	0.0000	0.0000	0.1686	0.0000	0.6685	0.0000	0.2553	0.0000	0.0770	0.0000	0.3365	0.0000	0.0003
α_1	0.0993	0.0000	0.0258	0.0001	0.0000	1.0000	0.0341	0.4465	0.0851	0.0000	0.0125	0.0032	0.0432	0.0324	0.0046	0.4732
β	0.7712	0.0000	0.8923	0.0000	0.8896	0.0000	0.8979	0.0000	0.8165	0.0000	0.9270	0.0000	0.9110	0.0000	0.8131	0.0000
γ	0.1245	0.0009	0.0883	0.0000	0.1700	0.0000	0.1035	0.0176	0.1295	0.0000	0.0968	0.0000	0.0490	0.0169	0.3334	0.0000
ξ	0.9834	0.0000	0.9477	0.0000	0.7783	0.0000	0.9603	0.0000	0.9509	0.0000	0.9205	0.0000	0.9621	0.0000	0.8552	0.0000
ν	5.6700	0.0000	9.5179	0.0000	9.9281	0.0000	5.9242	0.0000	4.4273	0.0000	9.2471	0.0000	4.5839	0.0000	5.6795	0.0000
LLH	6,958.97		8,008.10		10,071.37		9,346.75		9,445.1		9,245.87		9,140.32		9,556.65	
Akaike	–5.0315		–5.7909		–7.2844		–6.7599		–6.8310		–6.6868		–6.6104		–6.9118	

Figure 2 shows the main tree of the C-Vine copula obtained when the sum of the absolute values of Kendall's Tau of all pairs of countries are maximised.

The root node is SP500 (USA); this tells us that it is America's leading financial stock market. The nodes that are directly related to the USA are Brazil, Mexico, and Canada. Each of these last nodes connects individually with Argentina, Chile, and Peru, respectively, and from Chile, Colombia follows it.

Figure 2 The main tree of the C-Vine copula is estimated. Kendall's Tau is on every edge (see online version for colours)



From here, we can obtain routes of contagion between the American financial markets. For example, a financial event that affects Chile would initially have direct consequences on Colombia and Mexico; from the latter, it passes to the USA and from there to Brazil and Canada, finally reaching Argentina via Brazil and Peru via Canada.

Table 4 presents the estimation of the vine copula with the optimal pairwise copulas on each edge. Each row presents the relationship that exists between each market, numbered from 1 to 8 (Argentina, Brazil, Canada, Chile, Colombia, Mexico, Peru, and USA); for example, 12 is the relationship between Argentina (1) and Brazil (2), 56|4 is the relationship between Colombia (5) and Mexico (6) conditional on performance in the Chilean market (4). The optimal copula obtained, the estimated parameters of the copula and their standard errors, Kendall's tau, and the dependency coefficients in upper and lower tails are shown.

The optimal copulas found in this study are Gaussian, Clayton, Gumbel, and Frank, determined by one parameter, and student t, BB1, and BB8, determined by two parameters.

The Gaussian copula and student t represent the normal elliptic copulas and student's t. Putting an S in front of it is the copula rotated 180 degrees.

Figures 3 and 4 show the dynamics of the conditional dependence measures for the bivariate density distributions of the principal C-Vine tree. These estimations are based on equation (4).

Table 4 Result of the estimation of the vine copula

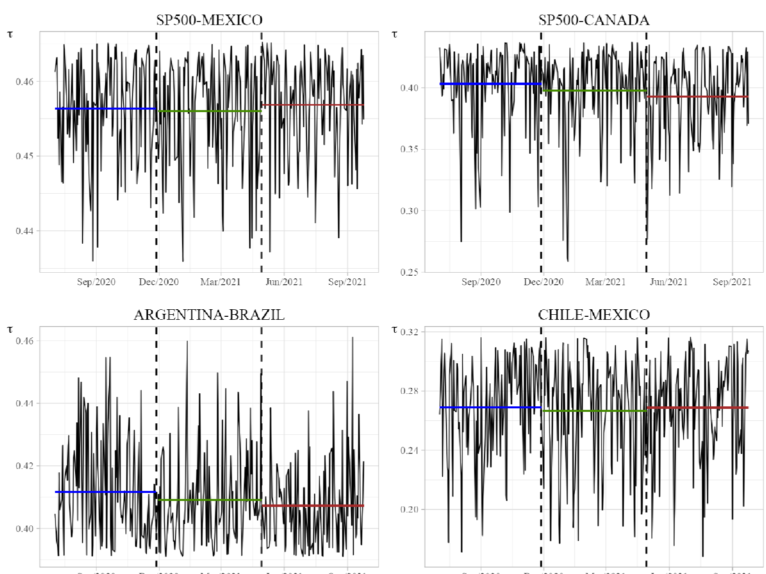
	Copula	*Parameter 1*	*Std. err.*	*Parameter 2*	*Std. err.*	τ	τ^L	τ^U
12	Student t	0.4425	0.0145	19.2982	7.1411	0.2918	0.0109	0.0109
54	Student t	0.3448	0.0168	13.1812	3.1222	0.2241	0.0197	0.0197
46	Student t	0.4297	0.0151	12.5015	2.7353	0.2828	0.0365	0.0365
73	SBB1	0.1952	0.0342	1.2903	0.0248	0.2939	0.2888	0.0638
38	Student t	0.7050	0.0088	10.0165	1.8886	0.4981	0.1948	0.1948
28	BB1	0.3114	0.0349	1.2453	0.0231	0.3052	0.1674	0.2553
68	Student t	0.5096	0.0134	11.3390	2.3610	0.3404	0.0677	0.0677
18|2	Frank	1.3691	0.1166	-	-	0.1488	-	-
56|4	Student t	0.2228	0.0181	30.0000	13.4588	0.1431	0.0001	0.0001
48|6	Gaussian	0.2052	0.0175	-	-	0.1316	-	-
78|3	Frank	0.6193	0.1158	-	-	0.0683	-	-
32|8	Student t	0.1939	0.0189	19.6986	7.5418	0.1242	0.0012	0.0012
26|8	Gaussian	0.2821	0.0168	-	-	0.1820	-	-
13|82	Frank	0.8730	0.1162	-	-	0.0960	-	-
58|64	Student t	0.1576	0.0196	10.4663	2.3342	0.1007	0.0142	0.0142
42|86	SBB8	1.8860	0.3345	0.7294	0.1150	0.1469	-	-
72|83	Student t	0.2053	0.0184	27.9913	12.3389	0.1316	0.0001	0.0001
36|28	Gaussian	0.1267	0.0184	-	-	0.0809	-	-
17|382	Frank	0.7429	0.1167	-	-	0.0818	-	-
52|864	Gaussian	0.1245	0.0185	-	-	0.0795	-	-
43|286	Student t	0.0988	0.0198	16.3142	5.9804	0.0630	0.0015	0.0015
76|283	Student t	0.1114	0.0195	18.5964	6.2633	0.0711	0.0008	0.0008
16|7382	Frank	0.5020	0.1170	-	-	0.0555	-	-
53|2864	BB1	0.0546	0.0266	1.0387	0.0127	0.0628	0.0000	0.0510
47|3286	Gaussian	0.1291	0.0184	-	-	0.0824	-	-
14|67382	Frank	0.4809	0.1165	-	-	0.0532	-	-
57|32864	Gumbel	1.0595	0.0122	-	-	0.0561	-	0.0763
15|467382	SGumbel	1.0205	0.0113	-	-	0.0201	0.0276	-

Figures 3 and 4 indicate three periods, before December 2020 (pre-covid), between December 2020 and April 2021 (covid), and after that date (post-covid). A horizontal line in each period represents the conditional probability mean in each period. Figure 3, according to Table 4, shows the optimal bivariate copula for the pairs USA-Argentina, USA-Canada, Argentina-Brazil, and Chile-Mexico is a student's t. In this copula, the upper and lower conditional probability are the same.

In this context, the estimation of the Student's t copulas indicates a high likelihood that an extreme increase (decrease) in a financial market will result in an extreme increase (decrease) in the other financial market, confirming the high dependence between the financial market returns. In all cases, there is a slight decrease in the conditional probability in the Covid period, interpreted as an internal adjustment. Then, the conditional probability returns to its level in the cases of USA-Mexico and Chile-Mexico or continues with the adjustment in the cases of Argentina-Brazil and USA-Canada.

Figure 4, according to Table 4, shows the optimal bivariate copula for the USA-Brazil and Canada-Peru. In these copulas, the upper and lower conditional probability are different in that the probability that in a market, an increase in one causes an increase in another is different from the one that a decrease in one causes a decrease in another.

Figure 3 Conditional tail dependences obtained by student t copula for the main C-Vine tree according to Table 4 (see online version for colours)



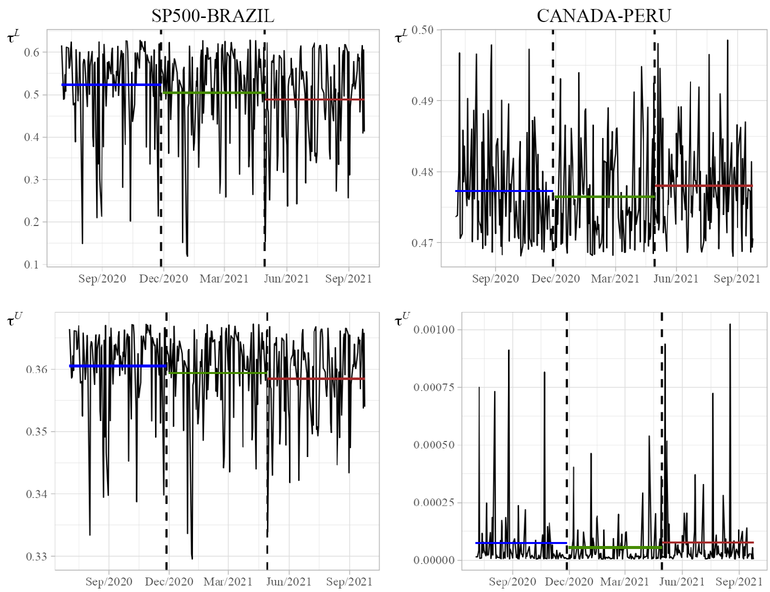
Notes: The mean for the pre-covid (blue), covid (green), and post-covid (brown) periods are shown.

The dependency measures are, again, slightly lower in the covid period.

The conditional dependency measure in the lower tail is more significant than in the upper tail. This indicates that there is a greater probability of having extreme negative returns in one stock market due to extreme negative returns in another stock market, such that the probability that in one market, an increase in one causes an increase in another is different from the that a decrease in one causes a decrease in another.

The greatest probability of having extreme positive returns in one market given extreme positive returns in another is between USA and Mexico; however, the greatest probability of having extreme negative returns in one market given extreme negative returns in another is between USA-Brazil, followed by Canada-Peru and USA-Mexico, therefore, in a period of crisis or phenomena such as covid, the contagion routes between American financial markets pass, mainly through the USA and from there it is transmitted to Argentina via Brazil, to Peru via Canada and to Chile and Colombia via Mexico.

Figure 4 Conditional tail dependences are obtained by BB1 copula (left) and SBB1 copula right for the main C-Vine tree, according to Table 4 (see online version for colours)



Notes: Lower conditional probability (up) and upper conditional probability are shown. The mean for the pre-covid (blue), covid (green), and post-covid (brown) periods are shown.

7 Conclusions and discussion

This document estimates the degree of dependence in six American stock markets: Argentina, Brazil, Canada, Chile, Colombia, Mexico, Peru, and the USA. Three study periods are considered: before, during, and after covid. A slight change was observed in the dependency levels in the three periods, but it is statistically not insignificant.

The marginal behaviour of the returns of the American stock indices is modelled by an AR(1)-TGARCH(1,1) process. Empirical results suggest:

1 that each of the analysed series of financial markets indexes returns can be adequately described with the proposed AR(1)-TGARCH model

2 that a leverage effect exists in returns: volatility increases when returns fall.

The previous leads us to affirm that modelling the behaviour of the marginal distributions for returns demonstrates, from a methodological point of view, the convenience of using conditional variance models to describe the behaviour of volatility, such as the TGARCH models. These estimated models describe, in all cases, the typical characteristics of financial time series and their volatility, such as clusters, leverage effect, excess kurtosis, and the possibility of considering the asymmetric effect by using a standardised skewed t-distribution for innovations.

3 There is a linear dependence between financial markets returns (as indicated by Pearson correlation) and a high degree of concordance, as shown by Kendall's tau

4 that there is a high degree of conditional dependence in the lower tail and significant degree of conditional dependence on the upper (right) tail.

This point leads us to conclude that there is a strong and stable probability of an increase (decrease) in an American financial return, following an increase (decrease) in other financial returns varying in time.

The findings provided evidence that America's primary financial stock market is the USA.

From this financial market, we can obtain routes of contagion between the American financial markets. Some phenomena affecting some American financial markets will inevitably pass through the American financial market and infect the other markets.

Considering conditional probabilities, we found, again, that the highest probability of having extreme negative returns in one market given extreme negative returns in another is between USA-Brazil, followed by Canada-Peru and USA-Mexico; therefore, in periods of crisis, the routes of contagion between the American financial markets pass, mainly through the USA, and from there, it is transmitted to Argentina via Brazil, to Peru via Canada, and to Chile and Colombia via Mexico.

Finally, in the post-covid period, the contagion routes remain, but with a lower degree of dependence, motivating possible diversification, although with the possibility of becoming infected again.

There is an evolution of the dependency structure captured by the copula approach, which leads to an effect or contagion route. As discussed, the American stock markets have a higher probability of extreme losses, so the dependency structure between them is more potent in periods of crisis. This implies that it becomes more challenging to diversify investment portfolios in the region in times of crisis.

Modelling stock returns and their dependency is helpful for making risk management, investment, and asset valuation decisions. In addition, including the copulas and considering the dependency parameters in tails allows us to describe better the joint behaviour of the returns that have implications for portfolio theory and risk analysis.

For possible subsequent studies, the possibility of extending the study and considering other markets or economic variables to measure the dependence between them is highlighted, as well as considering other types of distributions, whether symmetric or asymmetric, for disturbances and different models for the volatility of returns or consider calendar effects.

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Notes

1 Tail dependence measures vary according to the equations that describe the tails' dynamics, but the copulas' functional form remains fixed.

2 The ARCH/GARCH family includes more than a hundred time-series models. The ARCH and GARCH acronyms are auto-regressive conditional heteroscedasticity and generalised auto-regressive conditional heteroscedasticity. These time-series models originate from the ones proposed by Engle (1982) and Bollerslev (1986).

3 The TGARCH acronym stands for threshold generalised autoregressive conditional heteroscedasticity.