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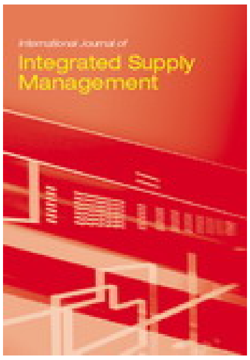

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Safety stock management in a two-stage supply chain with controllable lead time and batch shipments: a technical note

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Abstract: The joint economic lot size (JELS) model with stochastic demand and controllable lead time has received quite some attention in the literature. If a lot is shipped in batches from the vendor to the buyer, subsequent shipments may be subject to different lead times. Establishing a common safety stock for all batch shipments may lead to an inflated stockout risk for early batch shipments and an unnecessarily high inventory level for late batch shipments. To alleviate this shortcoming, a few authors proposed JELS models that adjust the safety stock level over the course of the inventory cycle. Existing works, however, do not always correctly establish the relationship between the vendor's and the buyer's inventory. This technical note adjusts the relationship between the buyer's and the vendor's inventory and numerically quantifies the error's impact on the expected average inventory as well as on the system's expected total cost.

Keywords: integrated inventory model; joint economic lot size; JELS; variable lead time; safety stock; backorder price discount; stochastic demand.

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1 Introduction

The joint economic lot size (JELS) model has attracted the attention of researchers for many years. Early works in this area are those of Goyal (1977) and Banerjee (1986), who studied a single vendor producing and delivering a single type of product to a single buyer. The objective of both papers was to develop a lot size policy that is optimal from a system point of view, instead of individually optimising the cost functions of the supply chain parties. These seminal papers have frequently been extended in the past, e.g., to the case of multiple buyers, (e.g., Joglekar and Tharthare, 1990; Banerjee and Banerjee, 1994) or vendors, (e.g., Chen and Sarker, 2010; Glock and Kim, 2014), to the case of more than two stages, (e.g., Banerjee and Kim, 1995; Kim and Glock, 2013), or to the case of stochastic demand, (e.g., Sharafali and Co, 2000; Ben-Daya and Hariga, 2004) or stochastic lead time (e.g., Sajadieh and Jokar, 2009; Sajadieh et al., 2009). We refer the reader to the review of Glock (2012) for an overview of the JELS literature.

Ben-Daya and Hariga (2004) proposed a JELS model with stochastic customer demand in which the vendor delivers the lot to the buyer in n shipments of size Q. The buyer was assumed to use a continuous-review, lot size-reorder point model to replenish its inventory. Glock (2009) extended the model to the case of unequal-sized batch shipments and Taleizadeh et al. (2010) to the case of multiple products.

In developing their proposed model, Ben-Daya and Hariga (2004) assumed that the lead time of a batch depends on a (lot size-dependent) production time and a non-productive transportation time. Hsiao (2008) argued that in this case, the lead time for the first batch is longer than the lead time for batches 2, …, n, as the production time of batches 2, …, n does not have to be considered in calculating the lead time of these batches. He therefore extended Ben-Daya and Hariga's (2004) model to account for two different reorder points, with the first one being valid for the first batch shipment and the second one for the remaining batches. The author assumed, however, that a single safety stock exists for the entire inventory cycle.

If we assume that subsequent shipments from a lot have different lead times, then the periods during which the system is at risk to suffer a stockout will be different for the shipments as well. Instead of establishing a single safety stock that is maintained over the entire inventory cycle, the company could establish a larger safety stock for those batches that suffer from an elevated stockout risk, and reduce the safety stock for batches associated with a shorter risk period. To the best of the authors' knowledge, multiple safety stocks in a JELS model were first discussed by Mou et al. (2017) and then by Sarkar and Giri (2022). However, Mou et al. (2017) incorrectly linked the vendor's and the buyer's inventory, leading to an incorrect expression of the expected total cost of the system. Sarkar and Giri (2022) adopted this error. This note thus proposes a correct formulation of the inventory trajectories and the expected total cost function of a single vendor-single buyer JELS model with batch shipments, stochastic demand, and multiple safety stocks. We adopt the scenario investigated by Sarkar and Giri (2022) for this purpose, but note that the fundamental relationships put forward in the following are valid for other supply chain environments as well.

2 A single vendor-single buyer JELS model with batch shipments, stochastic demand, and multiple safety stocks

Sarkar and Giri (2022) investigated a JELS model with a single vendor and a single buyer and assumed that demand is stochastic, lead time controllable, and that a backorder price discount exists. The lot produced by the vendor was assumed to be delivered in n equal batches of size Q to the buyer. The lead time for the first batch was modelled as the sum of the production time of the first batch, Q/P, with P being the production rate, and the transportation time, T. For batches 2, …, n, the lead time was set equal to the transportation time T. Note that this lead time formulation immediately follows from the assumption $P > D$, where D is the average demand rate, and from the assumption that the vendor produces and/or ships a batch only when ordered by the buyer. The first assumption is necessary to ensure that the vendor is able to meet the customer demand, and the second assumption reduces stock holding cost in the system, as keeping inventory is more expensive at the buyer than at the supplier. In this scenario, the lead time of the first batch consists of the production time Q/P and the transportation time. Then, when the remaining shipments are requested by the buyer, the vendor has already accumulated sufficient inventory to meet the buyer's order and can immediately deliver the batch. Hence, the lead time for batches 2, …, n coincides with the transportation time. The authors further assumed that two replenishment cycles with different lead times have different safety stocks.

In developing their proposed model, the authors incorrectly linked the vendor's inventory to the buyer's inventory, though. Both in their Figure 2 (please refer to their paper for this figure) and in the vendor's inventory carrying cost included in their equation (15), they assumed that the vendor delivers batches, on average, every Q/D units of time to the buyer. This is not correct, as it does not correctly reflect the buyer's expected inventory dynamics, which includes, by assumption, two different safety stocks between the first replenishment cycle and the other 2, …, n cycles. We will denote by S_1 the safety stock of the first replenishment cycle and by S_2 the safety stock of replenishment cycles 2, …, n.

Assume that because of the longer lead time of the first batch, the buyer wishes to establish a larger safety stock S_1 for this batch. To build up the additional safety stock, the buyer has to use a higher reorder point r_1, which leads to an earlier issue of the order at the vendor. The expected cycle time of this batch consequently is:

$$Z_1 = \frac{\left[Q - (S_1 - S_2)\right]}{D}. \tag{1}$$

For the remaining batches 2, …, n, the buyer reduces the safety stock to the lower lever S_2 again. This implies that, in addition to the batch size Q, the buyer also consumes the difference between both safety stocks, $S_1 - S_2$, in this cycle, leading to a larger expected cycle time for the second batch:

$$Z_2 = \frac{\left[Q - (S_2 - S_1)\right]}{D}. \tag{2}$$

The cycle times for the remaining batches 3, …, n are, as in the work of Sarkar and Giri (2022), Q/D. The expected total cycle time over replenishment cycles 1, …, n can now be calculated as follows:

$$Z_{\text{tot}} = \frac{\left[Q-(S_1-S_2)\right]}{D} + \frac{\left[Q-(S_2-S_1)\right]}{D} + \frac{Q}{D}(n-2) = n\frac{Q}{D}. \tag{3}$$

It can be observed that the expression of Z_{tot} is valid independently of the relationship between S_1 and S_2, i.e., independently of whether $S_1 \le S_2$ or $S_1 > S_2$. Figure 1 introduces the correct inventory patterns for buyer and vendor for the case where two different safety stocks are used on the buyer's side.

To establish the expected average inventory carrying cost for the buyer, we first calculate the area below the inventory curves in the lower part of Figure 1, which gives:

$$U = \frac{\left[Q-(S_1-S_2)\right]}{D}\left[\frac{Q-(S_1-S_2)}{2}+S_1\right] + \frac{\left[Q-(S_2-S_1)\right]}{D}\left[\frac{Q-(S_2-S_1)}{2}+S_2\right] + (n-2)\frac{Q}{D}\left(\frac{Q}{2}+S_2\right). \tag{4}$$

The expected average inventory of the buyer can now be calculated by dividing equation (4) by the expected total cycle time given in equation (3):

$$\overline{I}_B = \frac{U}{Z_{\text{tot}}} = \frac{Q}{2} + \frac{(n-1)}{n}S_2 + \frac{S_1}{n}. \tag{5}$$

The expected average inventory of the vendor can be calculated from Figure 1, for example by using the method proposed by Joglekar (1988), as follows:

$$\begin{aligned}\overline{I}_V &= \left\{\frac{nQ}{2}\left\{2\left\{\frac{Q}{P}+\frac{\left[Q-(S_2-S_1)\right]}{D}+(n-2)\frac{Q}{D}\right\}-\frac{nQ}{P}\right\}\right. \\ &\quad \left. -\left\{Q\frac{\left[Q-(S_2-S_1)\right]}{D}+\overbrace{2\frac{Q^2}{D}+\cdots+(n-1)\frac{Q^2}{D}}^{n-2\text{ terms}}\right\}\right\} \\ &\quad \frac{D}{nQ} = \frac{Q}{2}\left[n-1-\frac{D}{P}(n-2)\right]+\left(\frac{n-1}{n}\right)(S_1-S_2).\end{aligned} \tag{6}$$

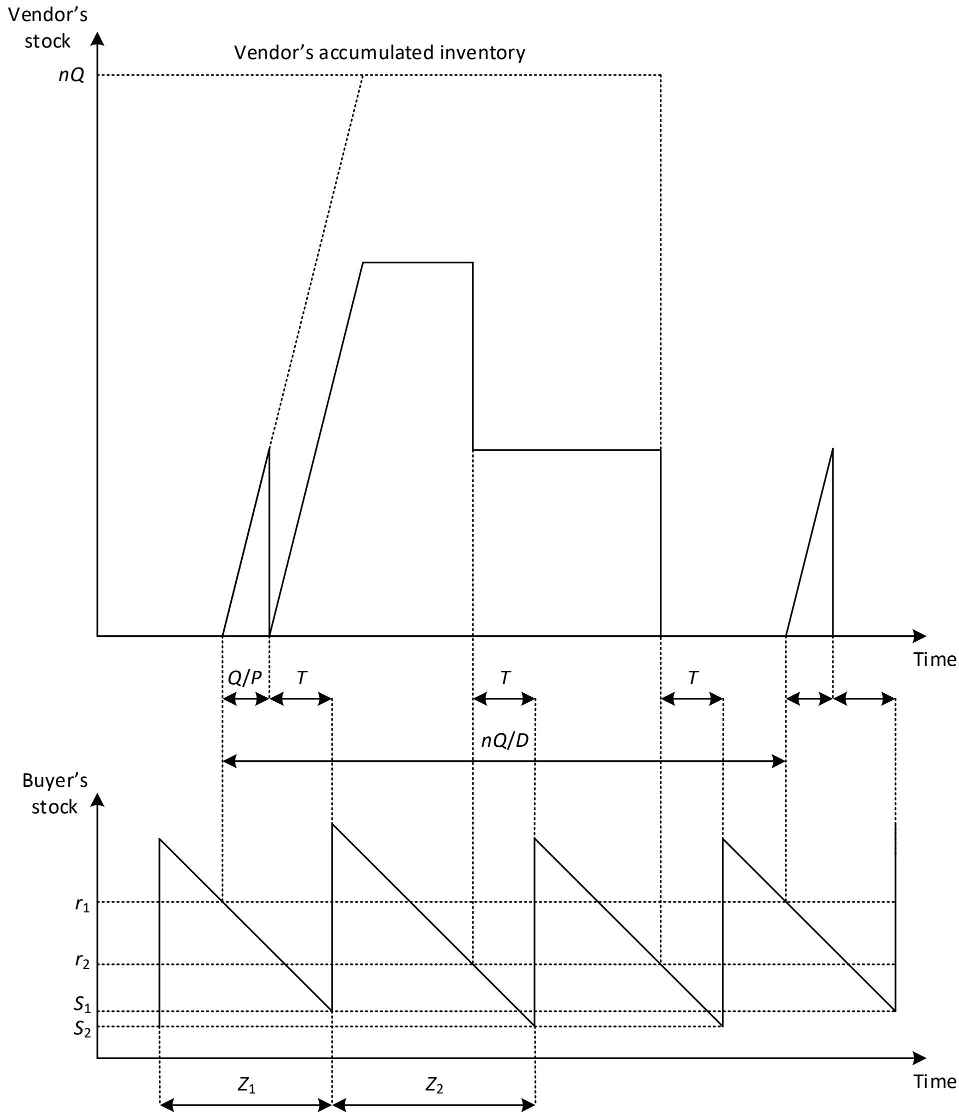
We now make the following observation. According to, e.g., Hadley and Whitin (1963), the safety stock in an inventory system controlled by means of a continuous review, (Q, r) policy in presence of full lost sales is

$$S = r - \mu + E\left[(X-r)^+\right], \tag{7}$$

where X is a random variable denoting the lead time demand and μ is its mean, and $E[X-r)^+]$ is the expected amount of lost sales per cycle. Consequently, if we let β be the fraction of shortage that is backordered, then the safety stock becomes

$$S = r - \mu + (1-\beta)E\left[(X-r)^+\right]. \tag{8}$$

Figure 1 Inventory time plots for a single vendor and a single buyer for the case of two different safety stocks at the buyer



If we now go back to the model of Sarkar and Giri (2022), we can note that the safety stock of the first replenishment cycle and the safety stock of replenishment cycles 2, …, *n* are incorrectly given by their equations (1) and (2), respectively. The correct expressions of S_1 and S_2 are, respectively,

$$S_1 = \kappa_1 \sigma \sqrt{L'} + \left(1 - \frac{b_x}{b_0} \frac{\delta}{1+\theta L'}\right) \sigma \sqrt{L'} \Psi(\kappa_1), \tag{9}$$

$$S_2 = \kappa_2\sigma\sqrt{L''} + \left(1 - \frac{b_x}{b_0}\frac{\delta}{1+\theta L''}\right)\sigma\sqrt{L''}\Psi(\kappa_2). \tag{10}$$

The expected total cost of the system per time unit can now be calculated as follows:

$$\begin{aligned}
\Re &= \Re(Q, \kappa_1, \kappa_2, b_x, T, n) \\
&= \frac{D}{nQ}\left[n(F + R(T)) + A_b + A_s\right] + H_b\left(\frac{Q}{2} + \frac{1}{n}S_1 + \frac{n-1}{n}S_2\right) \\
&\quad + H_v\left\{\frac{Q}{2}\left[n - 1 - \frac{D}{P}(n-2)\right] + \left(\frac{n-1}{n}\right)(S_1 - S_2)\right\} \\
&\quad + \frac{D}{nQ}\left\{\left[(b_x - b_0)\frac{b_x}{b_0}\frac{\delta}{1+\theta L'} + b_0\right]\sigma\sqrt{L'}\Psi(\kappa_1)\right. \\
&\quad \left. + (n-1)\left[(b_x - b_0)\frac{b_x}{b_0}\frac{\delta}{1+\theta L''} + b_0\right]\sigma\sqrt{L''}\Psi(\kappa_2)\right\},
\end{aligned} \tag{11}$$

where S_1 and S_2 are given by equations (9) and (10), respectively.

3 Numerical experiments

Let $\hat{I}_V = \bar{I}_V - \left(\frac{n-1}{n}\right)(S_1 - S_2)$, with $\bar{I}_V$ given by equation (6), i.e., $\hat{I}_V$ is the expected average inventory of the vendor in the model of Sarkar and Giri (2022). Let $\mathbf{v}_o = (Q_o, \kappa_{1,o}, \kappa_{2,o}, b_{x,o})$ be the vector minimising $\Re$, for a fixed pair $\phi = (n, T)$, and let $\hat{I}_{V,o} = \hat{I}_V(Q_o), \bar{I}_{V,o} = \bar{I}_V(\mathbf{v}_o)$ and $\Re_o = \Re(\mathbf{v}_o)$, all evaluated in correspondence to ϕ. Let $\mathbf{v}_M = (Q_M, \kappa_{1,M}, \kappa_{2,M}, b_{x,M})$ be the vector minimising the cost function of Sarkar and Giri (2022) for fixed ϕ. Let $\Re_M = \Re(\mathbf{v}_M)$.

We carried out numerical experiments to investigate the magnitude of the error made by Sarkar and Giri (2022) in evaluating the vendor's expected average inventory. These experiments were performed considering an inventory system with the same data as used by Sarkar and Giri (2022) in their Example 1, specifying that one year is the reference time unit, with 46 weeks per year and seven days per week, and that one dollar is the reference monetary unit. We assume that the values of the stockholding cost rate adopted by Sarkar and Giri (2022) are expressed in [$/unit/year] and that the value given for the standard deviation of the demand rate is relevant for one week (these aspects were not fully clear in their paper). Transportation time data are taken from Sarkar and Giri (2022), too.

To evaluate the impact of the error in the model of Sarkar and Giri (2022) on the system's behaviour, we introduce the following error metrics:

$$\text{PE}_{C1} = \frac{\Re_M - \Re_o}{\Re_o} \times 100, \tag{12}$$

$$\text{PE}_I = \frac{\hat{I}_{V,o} - \bar{I}_{V,o}}{\bar{I}_{V,o}} \times 100. \tag{13}$$

The results of these experiments are shown in Tables 1 and 2. Note that we considered values of n larger than 1 because, for $n = 1$, both models give the same result.

We can first observe that the model of Sarkar and Giri (2022) leads to a substantial error compared to the correct model presented in this paper. The percentage error regarding costs, PE_{C1}, ranges from a minimum value of 2.44 to a maximum value of 11.67. Moreover, PE_{C1} becomes larger for higher values of n. Increasing values of T give a larger PE_{C1}.

The percentage error made by Sarkar and Giri (2022) in evaluating the expected average inventory of the vendor, PE_I, ranges from a minimum of 5.26 to a maximum of 160.01. We also observe that PE_I shows a similar behaviour than PE_{C1} with respect to changes in n and T. It is noteworthy that the expected average inventory of the vendor, corresponding to the optimal solution for a fixed ϕ, is overestimated in the model of Sarkar and Giri (2022). In other words, in all considered problem instances, we have $S_1 < S_2$ or, equivalently, $\kappa_1 < \kappa_2$.

Table 1 Percentage error between $\Re_M$ and $\Re_o$ for different values of T and n

	PE_{C1}								
	n								
T [day]	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
56	3.14	5.45	7.11	8.35	9.32	10.09	10.72	11.23	11.67
42	2.92	5.10	6.66	7.82	8.72	9.43	10.00	10.47	10.86
28	2.63	4.64	6.07	7.12	7.92	8.54	9.04	9.44	9.77
21	2.44	4.32	5.66	6.63	7.36	7.92	8.36	8.71	8.99

Table 2 Percentage error between $\hat{I}_{V,o}$ and $\overline{I}_{V,o}$ for different values of T and n

	PE_I								
	n								
T [day]	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
56	42.66	78.60	104.85	123.74	137.18	146.61	153.08	157.36	160.01
42	28.48	55.54	74.83	88.43	98.01	104.73	109.38	112.53	114.58
28	13.36	32.70	46.36	55.91	62.62	67.37	70.73	73.08	74.71
21	5.06	20.69	31.74	39.42	44.83	48.67	51.42	53.38	54.78

3.1 Additional experiments

In this subsection, we present numerical experiments carried out to compare the model we developed with a simpler model that originates from it, and that is obtained assuming two different reorder points, but a single safety stock, as in Hsiao (2008). Equating S_1 with S_2 [see equations (9) and (10)], we get, e.g., κ_2 as a function of κ_1 (note that this equation can only be solved numerically). Then, substituting this value of κ_2 in equation (11), and setting $S_1 = S_2$ in the same equation, we obtain the cost model for the case with different reorder points, but identical safety stocks.

Let $\mathbf{v}_H = (Q_H, \kappa_{1,H}, b_{x,H})$ be the minimum-cost solution for the model with identical safety stocks for a fixed ϕ, and let $\Re_H$ be the corresponding minimum cost. The comparison is made assuming the same problem instance as in the previous experiments and considering the following quantity:

$$\text{PE}_{C2} = \frac{\Re_H - \Re_o}{\Re_o} \times 100. \tag{14}$$

The results are given in Table 3 and demonstrate the benefits originating from the adoption of different safety stocks. In fact, the model with identical safety stocks results in a higher cost than the one with different safety stocks. In the considered instance, the percentage cost difference spans from 0.03% to 9.07%, and it increases with T and n.

Table 3 Percentage difference between $\Re_H$ and $\Re_o$ for different values of T and n

	PE_{C2}								
	n								
T [day]	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
56	0.86	2.41	3.88	5.15	6.21	7.11	7.87	8.51	9.07
42	0.52	1.76	3.05	4.19	5.17	6.00	6.71	7.32	7.84
28	0.16	0.97	1.96	2.91	3.75	4.48	5.11	5.66	6.13
21	0.03	0.52	1.28	2.06	2.79	3.43	3.99	4.48	4.91

4 Conclusions

Despite the popularity JELS models with stochastic demand have enjoyed in the past, only a few authors acknowledged that an adjustment of the safety stock during the inventory cycle can lower the expected total cost of the supply chain. The few works that exist in this area incorrectly linked the buyer's and the vendor's inventories, however, leading to an incorrect approximation of the expected total system cost. The work at hand provided the correct expression for the expected average inventory of the vendor to correct these inconsistencies.

In numerical experiments, we estimated the magnitude of the error in the model of Sarkar and Giri (2022) both with respect to the minimum cost and the expected average inventory of the vendor. Depending on the problem setting, the numerical experiments showed that the error in the minimum cost can exceed 10%, and that the error in correctly estimating the average inventory of the vendor can be well above 100%. In a comparison of our model to an alternative setup with different reorder points, but a common safety stock for all batch shipments, we found that using multiple safety stocks can reduce the total cost by up to 9%, again depending on the parameter settings.

Future research could further investigate how adjustments of the safety stock over time affect both the service level and the expected total costs of a supply chain. Interesting scenarios to investigate would be the case where products deteriorate, such that items in the safety stock need to be replaced from time to time, or the case where inventory is subject to shrinkage. We leave these and other extensions of our work for future research.

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