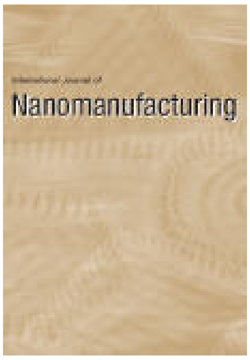

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Abstract: This paper presents fly cutting surface profile mathematical model using the spindle and guideways kinematic motion errors, as well as cutting parameters. Surface residual height model was firstly established using the feeding velocity, cutting spindle rotational speed and tool tip radius, it indicates that the cutting depth will not affect the surface geometric profile. Surface profile dispersion was carried out using cutter spindle rotation speed and guideways feeding velocity parameters. Guideways kinematic motion errors were introduced into the surface profile model through overlying method, and cutter spindle axial runout error was also introduced by filtering process using filtering convolution operations between the tool tip window filter and guideways kinematic overlapped surface profile. Their mathematical model expressions and illustrations were given, respectively. They were coherent with the cutting experiments results. The proposed model could be used for the surface profile prediction and machine tool error budget.

Keywords: fly cutting; kinematic motion error; process parameters; surface profile; overlying; convolution.

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1 Introduction

Fly cutting process was the relative movement between the single point diamond tool and workpiece, it was generally used to manufacturing non-ferrous workpiece. As the single diamond cutting was deterministic process, cutting surface profile accuracy and roughness value influence factors could be identified and evaluated (Zhang et al., 2016a; Brinksmeier et al., 2017). Influence of the spindle vibration towards fly cutting surface finish was investigated theoretically and experimentally, it indicates that the spindle vibration was the major factors influence of the surface roughness (Xi et al., 2020). Based on the cutting heat-generation and thermal transfer theory, temperature field model was established to guide the selection of cutting parameter and tool tip for the KDP fly-cutting process (Li et al., 2021). The fly-cutting tool flank wear and its influences on surface roughness in ultra-precision raster fly cutting was investigated through examination of cutting chip morphologies, and a mathematical model was established to identify the width of flank wear land and the theoretical surface roughness under tool flank wear effects (Brinksmeier et al., 2017). Cutting chips and size effect were also used to evaluate the raster milling surface quality (Zhang et al., 2016b; Chen et al., 2017). Based on the assumption that size effect would affect the formation of surface roughness pattern in fly-cutting process, the optimisation strategy of cutting parameters for the multiphase alloy was proposed (Zhang et al., 2020). Spindle error influence towards surface frequency domain error formation was investigated using analysis of two different evaluation directions of the machined surface profile, and relationship between the surface topography and spindle dynamic performance was established using simulation and experiment method (Yang et al., 2016; An et al., 2018). Dynamic error of spindle was tested using five-capacitance displacement sensors to research waviness errors along feed-direction on fly cutting surfaces (Miao et al., 2017). Additionally, external aerodynamic forces caused the air flow between the tool holder and workpiece was studied using CFD and experiment analysis (Kong and Cheung, 2012). Fly cutting process for hard ceramics was also studied, according to the simulation and experiments results, the crack-free nanometre-level surface roughness could be obtained by controlling the chip thickness less than 1 μm for zirconia ceramics (Deng et al., 2019). The error budget methodology for designing and characterising machines used to manufacture or inspect parts with spatial frequency-based specifications was established (Sun et al., 2015). A novel surface analytical model for cutting linearisation error in fast tool/slow slide servo diamond turning was built based on the surface analytical model and cutting linear error analysis (Neo et al., 2015).

This paper proposes fly cutting surface profile mathematical model. Cutter spindle rotation speed and workpiece linear feeding velocity were used to disperse the surface, number of the transverse discrete nodes was determined by the ratio of spindle rotary speed and feeding velocity, while the longitudinal number was calculated by the roughness sampling length. Spindle runout error and guideway straightness would overly onto the dispersed surface, and they were compounded together thereafter. The final cut surface could be obtained through the window filtering between the tool tip filter and the previous compound surface. The proposed model would be used to evaluate the surface profile influence factors, predict profile accuracy and guide the fly cutting machine design.

2 Geometry cutting model for surface profile

The fly cutting process was illustrated in Figure 1, it consists of cutter tool spindle and workpiece feeding guideways. The single point diamond tool was fixed onto the tool holder that could be aligned in two directions for the cutting angle's subtle adjustment, and the cutter has a circular arc. Tool cutter counterbalance was arranged on the opposite side of tool disk for the rotation imbalance reduction. The cutter spindle uses the aerostatic bearings of porous restrictors, the torque motor was adopted to drive the shaft, high speed magnetic encoder supplied the speed feedback signal. The linear guideways also adopted the aerostatic bearings while the linear motor was used to drive the carriage, the enclosed grating sensor with a resolution of 50nm was equipped to supply the displacement feedback signal.

Figure 1 Illustration of fly cutting process (see online version for colours)

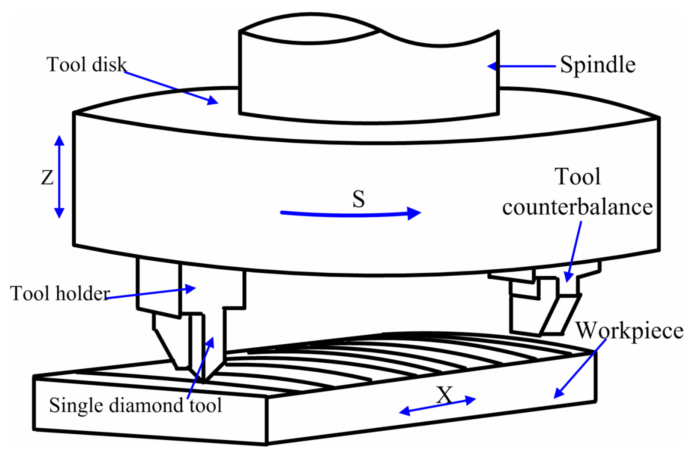
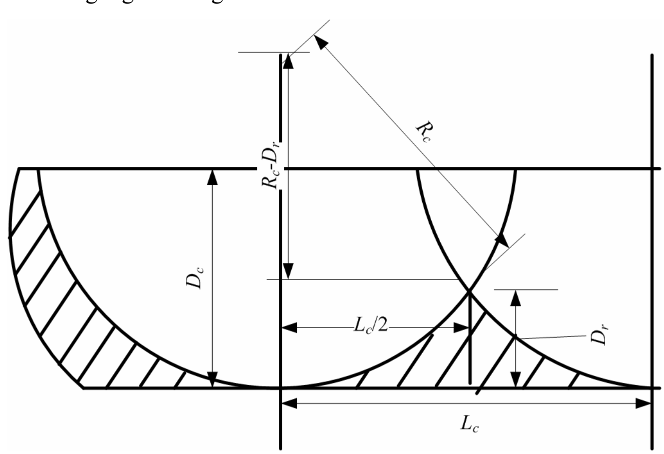


Figure 2 Residual height generating model

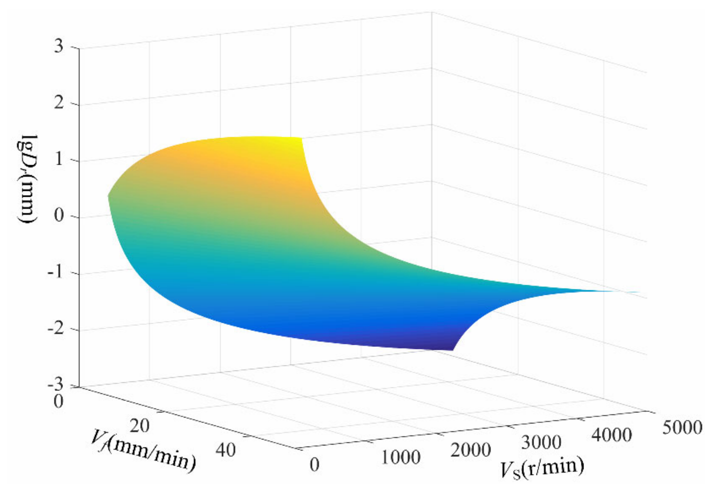


Fly cutting surface profile was generated through the tool sweeping movement across the workpiece surface, as shown in Figure 2, D_c, L_c and R_c are cutting depth, feed step size and tool radius, respectively. According to the geometric relationship illustrated in Figure 2, the residual height D_r could be expressed as equation (1) without considering the kinematic motion errors.

$$D_r = f\left(D_c, f_c, R_c\right) = R_c - \sqrt{R_c^{\,2} - \left(L_c/2\right)^2} = R_c - \sqrt{R_c^{\,2} - \left(V_f/2n_s\right)^2} \tag{1}$$

The feed step size L_c is the ratio of the feeding velocity V_f to the cutting spindle rotation speed n_s. According to equation (1), surface residual height was illustrated in Figure 2, tool tip radius R_c equals to 100 mm. According to the simulation results, as shown in Figure 3, the cutting surface profile could be controlled under 10 nm, however, the actual cut surface profile error is barely less 20 nm. According to equation (1), surface residual height model could be used to optimise the cutter spindle speed and feeding velocity selection instead of comprehensively predicting the actual cut surface profile, and the cutter spindle and guideways motions errors need to be taken into consideration to thoroughly investigate the surface profile generation mechanism.

Figure 3 Surface residual height simulation map (see online version for colours)



3 Mathematical model using cutting parameters and kinematic motion errors

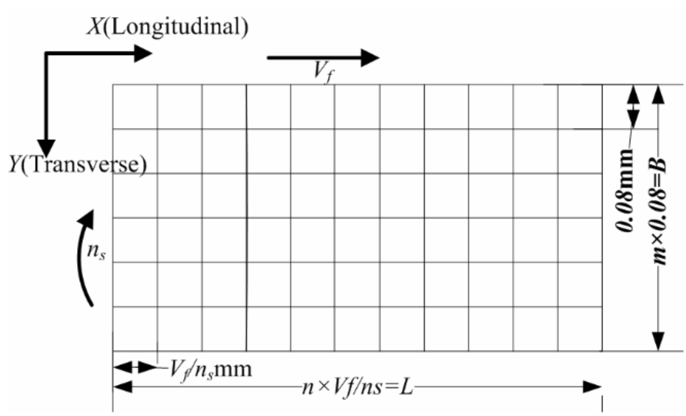
Cutter spindle and guideway kinematic motion errors, cutting parameters and tool tip would affect the cut surface profile whose mathematical model could be built using surface dispersion, kinematic motions error overlying and tool tip filtering method.

3.1 Surface dispersion using cutting parameters

To establish the surface profile mathematical model, the surface needs to be represented numerically, fly cutting surface was dispersed into numerous nodes firstly, and the kinematic motion errors would overly onto this dispersed surface thereafter. The longitudinal nodes number in the guideway feeding direction equals to the residual height number which is determined by the cutter spindle rotation speed ns and feeding velocity V_f, while the transverse interval is 0.08 mm determined by the roughness sampling length as shown in Figure 4. The longitudinal number M and transverse number N could be determined using equation (2) for the given workpiece with length L and width B.

$$\begin{cases} M = L * n_s / V_f \\ N = B / 0.08 \end{cases} \tag{2}$$

Figure 4 Surface dispersion illustration



3.2 Kinematic motion errors overlying and tool tip filtering

Kinematic motion error measurement is generally 1-D, before they are overlaid onto the dispersed surface, 2-D expansion of the kinematic motion errors needs to be done. And the 'interp1' and 'interp2' functions of MATLAB could be used to generate the kinematic motion node error surface with the grid points corresponding to the nodes determined by the cutter spindle rotation speed and guideways feeding velocity.

3.2.1 Guideway straightness and overlying

Guideways straightness in Z direction was prominent for the surface roughness, it was tested using straightness edge and TESA LVDT inductive sensor, and the error map was illustrated in Figure 5, straightness error curve $g(x)$ is function of the workpiece feeding step size x. Straightness would overly onto transverse line during cutting process as shown in Figure 6. MATLAB function 'interp1' was used to generate the error line function $g(x)$, and equation (3) was derived using impulse sampling. As each longitudinal line was the same for guideways straightness surface overlying, it could be expressed as equation (4) and illustrated as Figure 7.

$$g(n) = \sum_{l=1}^{N} g(k) * \delta(n-l) \quad (l = 1, 2, 3, \dots, N) \tag{3}$$

$$g(m,n)=\sum_{k=1}^{M}\sum_{l=1}^{N} g(k,l)*\delta(m-k,\,n-l) \quad (m=1,2,3,\ldots,M;\, n=1,2,3,\ldots,N) \tag{4}$$

Figure 5 Guideways line straightness (see online version for colours)

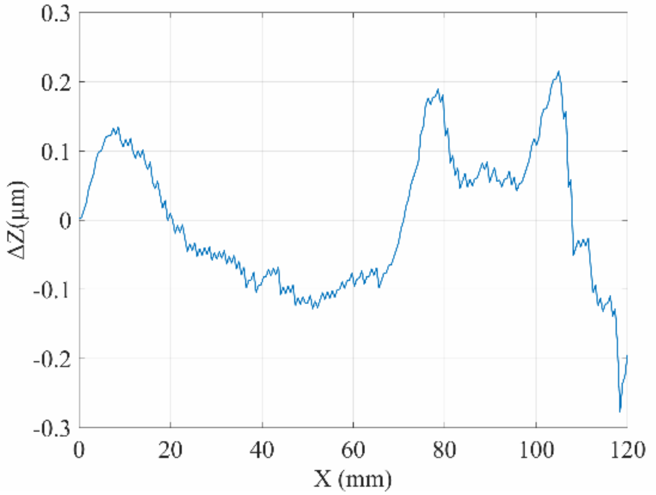


Figure 6 Guideways straightness line overlying map (see online version for colours)

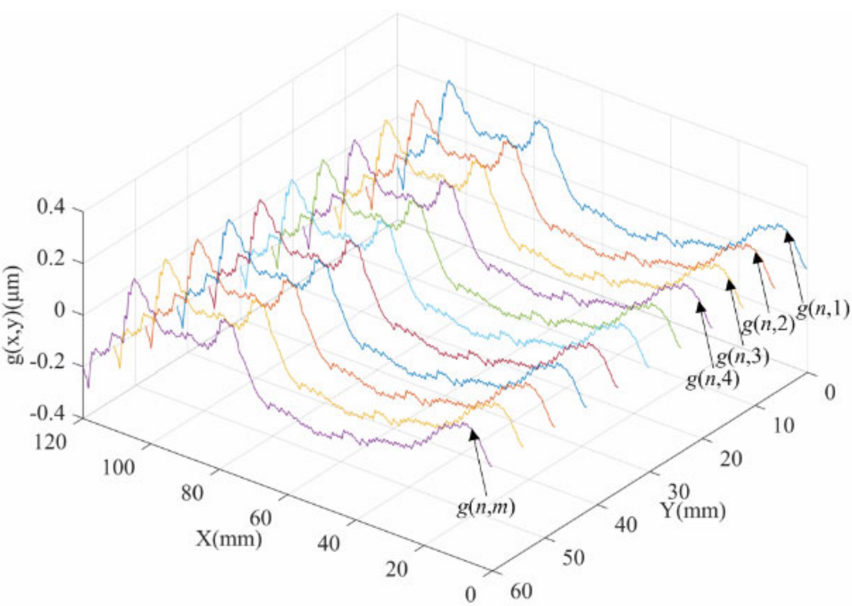
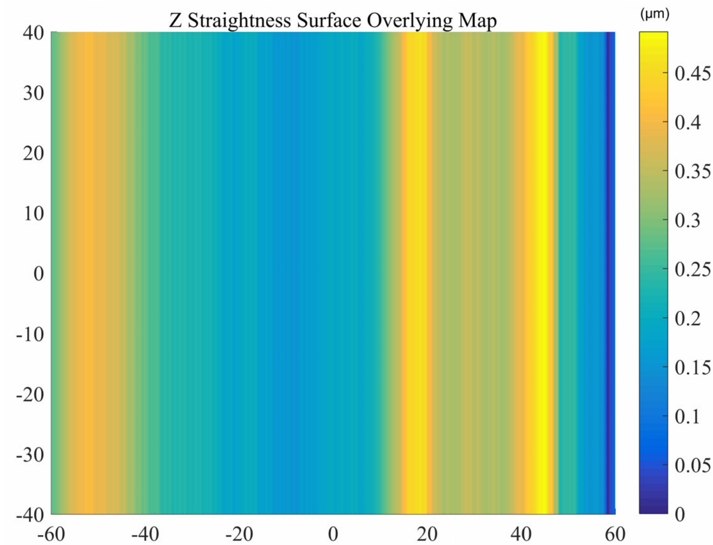


Figure 7 Guideways straightness surface overlying map (see online version for colours)



3.2.2 Tool tip filtering using cutter spindle axial runout

Cutter spindle axial runout influence is prominent comparing with the radial runout, and it was measured using the quartz standard sphere and TESA LVDT. The error map was illustrated in Figure 5, axial runout error $f(\psi)$ is function of the spindle rotary angle ψ. Axial runout would overly onto transverse line during cutting process as shown in Figure 6. MATLAB function 'interp1' was used to generate the error line function $f(y)$ based on the axial runout error illustrated in Figure 7, and equation (3) was derived using impulse sampling for the discrete representation of the axial runout line overlying. As each transverse line was the same, axial runout surface overlying could be expressed as equation (4) using the impulse sampling method.

$$f(m) = \sum_{k=1}^{M} f(k) * \delta(m-k) \quad (m = 1, 2, 3, \ldots, M) \tag{5}$$

Unlike the guideways straightness, cutter spindle axial runout error was introduced into the surface profile through the cutting process. As the single diamond tool sweeps across the surface, it could be viewed as the filtering process. Single diamond cutting tool was the window filter function $H(m, p)$ which could be expressed as follows:

$$H(m, p) = f(m)U[p] \Big/ \sum_{i=1}^{M} U[m] \tag{6}$$

where the index M is transverse number, and P is determined by the longitudinal interval and window filter width, it could be expressed as follows. The width of the window filter function $U(p)$ is 3 mm that is equal to the single point diamond tool tip width, and the filter amplitude $f(m)$ of equation (5) is the cutter spindle axial runout error as illustrated in Figure 8, and the cutting process could be expressed as the convolution filtering between the kinematic motion overlaid surface $g(m, n)$ and the cutting tool filter $H(m, n)$. The final cut surface profile mathematical model could be expressed as equation (7).

$$S(m,n) = C(m,n) * H(m,n) \tag{7}$$

Figure 8 Cutter spindle axial runout (see online version for colours)

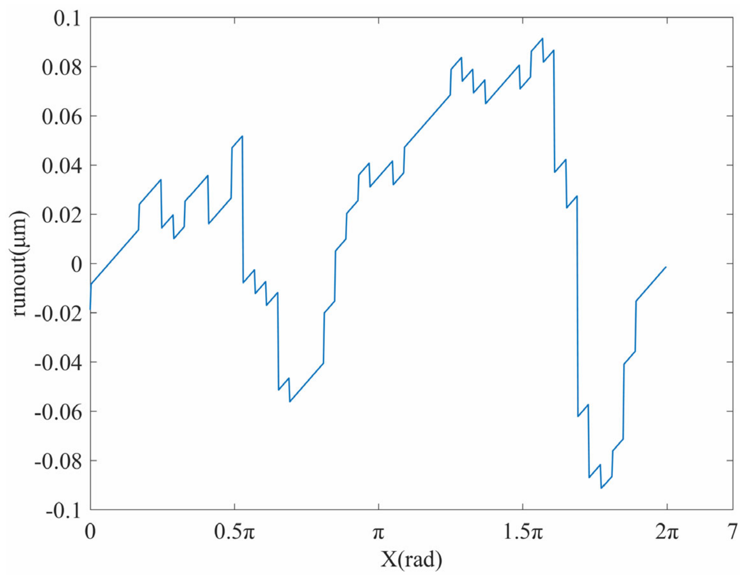
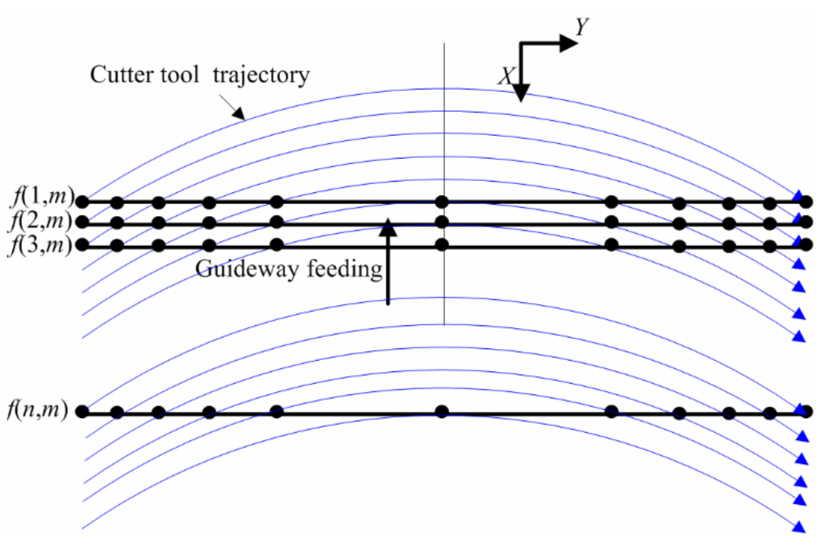


Figure 9 Spindle axial runout line overlying map (see online version for colours)

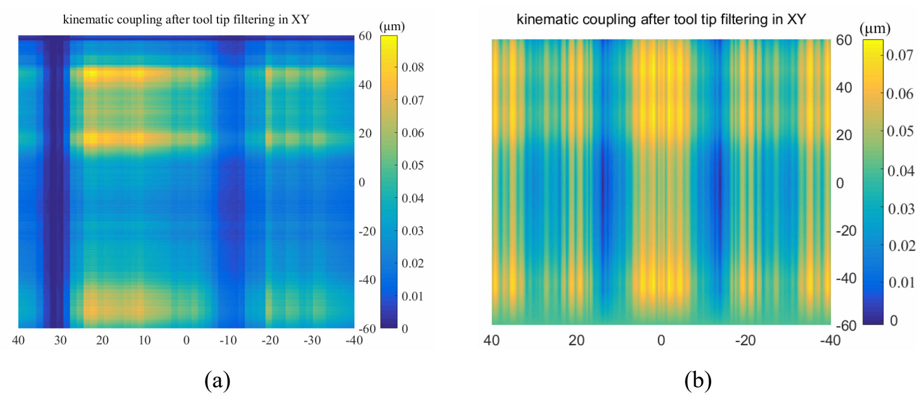


The final cut surface profile was simulated according to equation (7) using the parameters listed in Table 1, and the surface profile was illustrated in Figure 10 with two sets of given cutting parameters as shown in Table 1.

Table 1 Parameters for the mathematical model and cutting experiments

Items	*Symbol*	*Parameters*	
Tool radius	R	100 mm	
Workpiece size	$L \times B$	120 × 80 mm	
Cutter spindle rotational speed	n_s	2,000 rpm	3,000 r/min
Guideways feeding velocity	V_f	10 mm/min	10 mm/min
Transverse nodes number	M	24,000	36,000
Longitudinal nodes number	N	1,000	1,000

Figure 10 Surface simulation after tool tip filtering (see online version for colours)



4 Fast fly cutting experiments and discussion

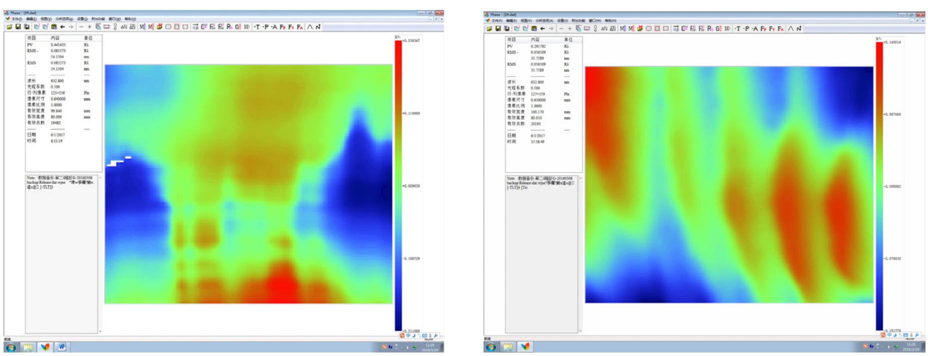
Using the parameters listed in Table 1, the model validating experiment was carried out on the developed fly cutting machine tool as shown in Figure 11, and its kinematic motion error was tested using the methods described in Section 3.

The cut profile surface was tested using plane phase shift interferometer, the measurement profile was illustrated in Figure 12, its P-V value were 0.441 μm and 0.291 μm while the simulation surface P-V values were 0.089 μm and 0.075 μm as shown in Figure 10, as the actual cutting process include the cutting vibration caused by the cutting force, tool tip roughness and room temperature variation, etc. However, the surface profile error distribution was coherent which suggested that the proposed mathematic model was quiet reliable and could be used to investigate the kinematic motion error influence, optimise the cutter spindle rotation speed and feeding velocity parameters.

Figure 11 Experimental machine tool and cutting picture, (a) fly cutting machine (b) cutting process (see online version for colours)

(a) (b)

Figure 12 Cut surface profile measurement result (see online version for colours)



(a) (b)

5 Conclusions

This paper explored the numerical relationship between the surface profile and machine tool's kinematic motion errors and cutting parameters for the fly cutting process, the specific conclusions from the study are as follows:

1 Surface dispersion method was proposed, and the cutter spindle rotation speed and guideways feeding velocity were used to determine transverse discrete nodes number while the longitudinal number was calculated by the roughness sampling length.

2 Guideways straightness error was introduced into the surface profile model by line and surface overlying method, and the final flyer cut surface mathematical model was obtained through the convolution operation between the straightness overlaid

surface profile and single diamond tool tip window filter whose amplitude was determined by the cutter spindle rotation axial runout value.

3 The proposed model was validated by comparing the cutting surface measured results and predicted model, and it is proved that proposed model could be used to predict the cut surface profile value and distributions.

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