Improved approximation of SISO and MIMO continuous interval systems

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Abstract: Recently, some engineering systems are modelled as interval systems. In this investigation, an improved approximation method is first presented for reducing the order of single-input-single-output (SISO) continuous interval systems and then the same method is extended for reducing the order of multi-input-multi-output (MIMO) continuous interval systems utilising multi-point Padé approximation. In contrast to traditional Padé approximants, multi-points are used for matching the response of higher order system to that of its model in case of multi-point Padé approximation. Matching around multi-points improves the overall quality of approximation. The multi-point Padé approximation is proposed for SISO system firstly. A test case for SISO system is considered to illustrate the efficacy of proposed method. Secondly, multi-point Padé approximation is extended for reducing the order of MIMO continuous interval systems. The proposed method is also investigated for one MIMO test case. From the results obtained for SISO and MIMO test cases, it is observed that multi-point Padé approximation is able to provide better approximants.

Keywords: interval systems; model reduction; multipoint Padé method; system modelling; SISO systems; MIMO systems.

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1 Introduction

The model reduction of large-scale interval systems is continued an important area of research in control system design (Choudhary and Nagar, 2018a, 2018b, 2018c). Model reduction not only reduces the order of large-scale interval systems but also offers simpler understanding of system, simpler controller design, reduced computational effort in simulation, etc. (Schilders et al., 2008; Singh et al., 2019).

Many good works are reported to lower the order of interval systems (Bandyopadhyay et al., 1994; Sastry et al., 2000; Yang, 2005; Kumar et al., 2011a, 2011b, 2011c, 2016; Saini and Prasad, 2010; Saraswathi et al., 2007; Choo, 2007; Choudhary and Nagar, 2013; Singh and Chandra, 2012; Anand et al., 2011; Gu and Yang, 2010; Singh et al., 2017). The pioneering work by Bandyopadhyay et al. (1994) derived the Routh-Padé approximation of continuous interval systems. In this work, the denominator of approximant is obtained using Routh table while the numerator is obtained by equating coefficients of power series expansions of large-scale interval system and approximant. Other important works based on Routh table includes large-scale interval system modelling using Routh approximation (Sastry et al., 2000), comments on Routh-Padé model reduction (Yang, 2005), modified Routh approximation combined with factor division method (Kumar et al., 2011a), Routh approximation with Cauer second form (Kumar et al., 2011c) and stable Routh-Padé approximation (Gu and Yang, 2010). Recently, Ismail (Ismail, 1996) has proposed multi-point Padé approximation for discrete interval systems. However, the reduction of single-input-single-output (SISO) interval system only is considered in Ismail (1996).

Inspired from (Ismail, 1996), in this contribution, a multi-point Padé approximation is proposed for model order reduction of higher order continuous interval systems. Further, the proposed method is extended for lowering the order of multi-input-multi-output (MIMO) continuous interval systems. In multi-point Padé approximation, first $2k$ points are chosen around which the responses of higher order continuous interval system and model are to be matched, where k is the order of model. In multi-point Padé approximation, $2k$ points are used for matching in contrast to traditional Padé approximation. Matching around multi-points improves the overall approximation. Firstly, the multi-point Padé approximation is proposed for SISO system. A test case for SISO system is also undertaken to illustrate the whole procedure. Secondly, the multi-point Padé approximation is extended for reduction of MIMO systems. The whole procedure of reduction of MIMO systems is carried out for one test system. The results obtained are also compared with some relevant works. The step and impulse responses are plotted for higher order system and its approximants.

The brief outline of the paper is as follows. Section 2 formulates the problem considered whereas Section 3 discusses the methodology to derive the approximant for higher order SISO continuous interval system. A test case for SISO system is provided in Section 4. Section 5 extends the proposed methodology for reduction of higher order MIMO continuous interval system. Further, a test case for MIMO system is provided in Section 6. Finally, the concluding remarks are carried out in Section 7.

2 Problem formulation

Consider a higher order continuous interval SISO system given by

$$G(s)=\frac{\left[\underline{b_0},\overline{b_0}\right]+\left[\underline{b_1},\overline{b_1}\right]s+\left[\underline{b_2},\overline{b_2}\right]s^2+\cdots+\left[\underline{b_{n-1}},\overline{b_{n-1}}\right]s^{n-1}}{\left[\underline{a_0},\overline{a_0}\right]+\left[\underline{a_1},\overline{a_1}\right]s+\left[\underline{a_2},\overline{a_2}\right]s^2+\cdots+\left[\underline{a_n},\overline{a_n}\right]s^n}=\frac{B(s)}{A(s)} \tag{1}$$

where $B(s)$ and $A(s)$ are numerator and denominator of system $G(s)$, respectively; and $[\underline{b_i},\overline{b_i}]$ for $(i=0,1,\dots,n-1)$ are interval coefficients of numerator with $\underline{b_i}$ as lower bound and $\overline{b_i}$ as upper bound, respectively. Similarly $[\underline{a_i},\overline{a_i}]$ for $(i=0,1,\dots,n)$ are interval coefficients of denominator with $\underline{a_i}$ as lower bound and $\overline{a_i}$ as upper bound, respectively.

Suppose, a *k*th order interval model of higher order system, given in (1), is desired. The transfer function of *k*th order interval model can be written as

$$R(s) = \frac{\left[\underline{d_0}, \overline{d_0}\right] + \left[\underline{d_1}, \overline{d_1}\right]s + \left[\underline{d_2}, \overline{d_2}\right]s^2 + \cdots + \left[\underline{d_{k-1}}, \overline{d_{k-1}}\right]s^{k-1}}{\left[\underline{e_0}, \overline{e_0}\right] + \left[\underline{e_1}, \overline{e_1}\right]s + \left[\underline{e_2}, \overline{e_2}\right]s^2 + \cdots + \left[\underline{e_k}, \overline{e_k}\right]s^k} = \frac{D(s)}{E(s)} \tag{2}$$

where $D(s)$ and $E(s)$ are numerator and denominator of *k*th order interval model $R(s)$, respectively. $[\underline{d_i}, \overline{d_i}]$ for $(i = 0, 1, \ldots k-1)$ and $[\underline{e_i}, \overline{e_i}]$ for $(i = 0, 1, \ldots, k)$ are, respectively, interval coefficients of numerator and denominator with $[\underline{e_0}, \overline{e_0}] = [1, 1]$.

3 Methodology

The *k*th order interval model, given in equation (2), is obtained from higher order interval system, provided in equation (1), using multi-point Padé approximation (Ismail, 1996; Lucas, 1993). In multi-point Padé approximation, the expansions of system and model are matched as

$$G(\lambda_j) = R(\lambda_j) \tag{3}$$

around $2k$ expansion points λ_j where $j = 1, 2, \ldots, 2k$. Contrary to Padé approximation where the expansion is matched around only two points, $s = 0$ and $s = \infty$, in multi-point Padé, the expansion is matched around $2k$ points, where k represents order of model. Due to matching around multiple points, the quality of approximation improves.

The $2k$ unknown parameters of equation (2) are obtained using equation (3). The formulation given in equation (3) becomes

$$P(s) = Q(s) \text{ at } s = \lambda_i \tag{4}$$

where $i = 1, 2, \ldots, 2k$ and

$$P(s) = D(s)A(s) \tag{5}$$

$$Q(s) = B(s)E(s) \tag{6}$$

The polynomials of equations (5) and (6) turn out to be, respectively, equations (7) and (8).

$$P(s) = \left[\underline{p_0}, \overline{p_0}\right] + \left[\underline{p_1}, \overline{p_1}\right]s + \left[\underline{p_2}, \overline{p_2}\right]s^2 + \cdots + \left[\underline{p_{n+k-1}}, \overline{p_{n+k-1}}\right]s^{n+k-1} \tag{7}$$

$$Q(s) = \left[\underline{q_0}, \overline{q_0}\right] + \left[\underline{q_1}, \overline{q_1}\right]s + \left[\underline{q_2}, \overline{q_2}\right]s^2 + \cdots + \left[\underline{q_{n+k-1}}, \overline{q_{n+k-1}}\right]s^{n+k-1} \tag{8}$$

where

$$
\begin{aligned}
&\left[\underline{p}_{n+k-1}, p_{n+k-1}\right] = \left[\underline{a}_{n-1}, a_{n-1}\right]\left[\underline{d}_{k}, d_{k}\right] \\
&\left[\underline{p}_{n+k-2}, p_{n+k-2}\right] = \left[\underline{a}_{n-2}, a_{n-2}\right]\left[\underline{d}_{k-1}, d_{k-1}\right] + \left[\underline{a}_{n-1}, a_{n-1}\right]\left[\underline{d}_{k-1}, d_{k-1}\right] \\
&\qquad\vdots \\
&\left[\underline{p}_{0}, p_{0}\right] = \left[\underline{a}_{0}, a_{0}\right]\left[\underline{d}_{k}, d_{k}\right] + \cdots + \left[\underline{a}_{n-1}, a_{n-1}\right]\left[\underline{d}_{0}, d_{0}\right]
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
&\left[\underline{q}_{n+k-1}, q_{n+k-1}\right] = \left[\underline{b}_{n-1}, b_{n-1}\right]\left[\underline{e}_{k}, e_{k}\right] \\
&\left[\underline{q}_{n+k-2}, q_{n+k-2}\right] = \left[\underline{b}_{n-2}, b_{n-2}\right]\left[\underline{e}_{k}, e_{k}\right] + \left[\underline{b}_{n-1}, b_{n-1}\right]\left[\underline{e}_{k-1}, e_{k-1}\right] \\
&\qquad\vdots \\
&\left[\underline{q}_{0}, q_{0}\right] = \left[\underline{b}_{0}, b_{0}\right]\left[\underline{e}_{k}, e_{k}\right] + \cdots + \left[\underline{b}_{n-1}, b_{n-1}\right]\left[\underline{e}_{0}, e_{0}\right]
\end{aligned}
\tag{10}
$$

A polynomial from $2k$ expansion points can be formed as

$$M(s) = \prod_{i=1}^{2k}\left(s - \bar{\lambda}_i\right) = s^{2k} + \gamma_{2k-1}s^{2k-1} + \cdots + \gamma_1 s + \gamma_0 \tag{11}$$

In multi-point Padé, Routh type array (Table 1) is formed using (7) and (11).

Table 1 Routh type array

$\left[\underline{p}_{n+k-1}, p_{n+k-1}\right]$	$\left[\underline{p}_{n+k-2}, p_{n+k-2}\right]$	$\underline{p}_{n+k-3}, p_{n+k-3}$	$\cdots$	$\left[\underline{p}_{0}, p_{0}\right]$
1	γ_{2k-1}	γ_{2k-2}	$\cdots$	γ_0
$\left[\underline{w}_{n+k-2}, w_{n+k-2}\right]$	$\left[\underline{w}_{n+k-3}, w_{n+k-3}\right]$	$\left[\underline{w}_{n+k-4}, w_{n+k-4}\right]$	$\cdots$	$\left[\underline{w}_{0}, w_{0}\right]$
1	γ_{2k-1}	γ_{2k-2}	$\cdots$	γ_0
$\left[\underline{x}_{n+k-3}, x_{n+k-3}\right]$	$\left[\underline{x}_{n+k-4}, x_{n+k-4}\right]$	$\left[\underline{x}_{n+k-5}, x_{n+k-5}\right]$	$\cdots$	$\left[\underline{x}_{0}, x_{0}\right]$
$\vdots$				
$\left[\underline{y}_{n+k-4}, y_{n+k-4}\right]$	$\left[\underline{y}_{n+k-5}, y_{n+k-5}\right]$	$\left[\underline{y}_{n+k-6}, y_{n+k-6}\right]$	$\cdots$	$\left[\underline{y}_{0}, y_{0}\right]$
1	γ_{2k-1}	γ_{2k-2}	$\cdots$	γ_0
$\left[\underline{z}_{n+k-5}, z_{n+k-5}\right]$	$\left[\underline{z}_{n+k-6}, z_{n+k-6}\right]$	$\left[\underline{z}_{n+k-7}, z_{n+k-7}\right]$	$\cdots$	$\left[\underline{z}_{0}, z_{0}\right]$

The first two rows of Table 1 are obtained from the coefficients of equations (7) and (11) while remaining rows are calculated as

$$
\begin{aligned}
\left[\underline{w}_{n+k-i-2}, w_{n+k-i-2}\right] &= \left[\underline{p}_{n+k-i-2}, p_{n+k-i-2}\right] - \gamma_{2k-i-1}\left[\underline{p}_{n+k-1}, p_{n+k-1}\right] \\
& \qquad i = 0,1,2,\ldots,(n+k-2) \\
\left[\underline{x}_{n+k-i-3}, x_{n+k-i-3}\right] &= \left[\underline{w}_{n+k-i-3}, w_{n+k-i-3}\right] - \gamma_{2k-i-1}\left[\underline{w}_{n+k-2}, w_{n+k-2}\right] \\
& \qquad i = 0,1,2,\ldots,(n+k-3) \\
&\vdots \\
\left[\underline{z}_{n+k-i-5}, z_{n+k-i-5}\right] &= \left[\underline{y}_{n+k-i-5}, y_{n+k-i-5}\right] - \gamma_{2k-i-1}\left[\underline{y}_{n+k-4}, y_{n+k-4}\right] \\
& \qquad i = 0,1,2,\ldots,(2k-1)
\end{aligned}
\tag{12}
$$

In similar manner, another Routh type array can be formed using equations (8) and (11). Finally, by equating the corresponding elements of the two arrays (first obtained for equations (7) and (11), and second obtained for equations (8) and (11)), the unknown coefficients of equation (2) are determined. The proposed methodology is explained with one test system considered in Section 4.

4 Test case for SISO system

In this section, one test system is provided to illustrate the proposed method.

Consider a second-order continuous interval system (Bandyopadhyay et al., 1994) with a transfer function

$$G(s) = \frac{[15,16]+[2,3]s}{[10,11]+[12,13]s+[2,3]\ s^2} = \frac{B(s)}{A(s)} \tag{13}$$

It is required to find a first-order model given as

$$R(s) = \frac{\left[\underline{d}_0, d_0\right]}{\left[\underline{e}_0, e_0\right]+\left[\underline{e}_1, e_1\right]s} = \frac{D(s)}{E(s)} \tag{14}$$

for the system, described in equation (13), using multi-point Padé approximation such that

$$\left[\underline{e}_0, e_0\right] = [1,1] \tag{15}$$

The following expansion points

$$s = 0, \quad s = 3 \tag{16}$$

are considered to obtain equation (14) which are the roots of the polynomial

$$M(s) = s^2 - 3s \tag{17}$$

For equations (13) and (14), the polynomials $P(s)$ and $Q(s)$, as given in equations (7) and (8), become

$$P(s) = \left[\underline{d}_0, d_0\right][2,3]s^2 + \left[\underline{d}_0, d_0\right][12,13]s + \left[\underline{d}_0, d_0\right][10,11] \tag{18}$$

$$Q(s) = \left[\underline{e}_1,\overline{e}_1\right][2,3]s^2 + \left\{\left[\underline{e}_1,\overline{e}_1\right][15,16] + \left[\underline{e}_0,\overline{e}_0\right][2,3]\right\}s + \left[\underline{e}_0,\overline{e}_0\right][15,16] \tag{19}$$

Table 1 becomes Table 2 for equations (17) and (18). Similarly for equations (17) and (19), Table 3 is obtained.

Table 2 Routh like array for $P(s)$ and $M(s)$

$\left[\underline{d}_0,\overline{d}_0\right][2,3]$	$\left[\underline{d}_0,\overline{d}_0\right][12,13]$	$\left[\underline{d}_0,\overline{d}_0\right][10,11]$
1	-3	0
$\left\{\left[\underline{d}_0,\overline{d}_0\right][12,13]+3\left[\underline{d}_0,\overline{d}_0\right][2,3]\right\}$	$\left[\underline{d}_0,\overline{d}_0\right][10,11]$	0

Table 3 Routh like array for $Q(s)$ and $M(s)$

$\left[\underline{e}_1,\overline{e}_1\right][2,3]$	$\left[\underline{e}_1,\overline{e}_1\right][15,16]+\left[\underline{e}_0,\overline{e}_0\right][2,3]$	$\left[\underline{e}_0,\overline{e}_0\right][15,16]$
1	-3	0
$\left\{\begin{matrix}\left[\underline{e}_1,\overline{e}_1\right][15,16]+\left[\underline{e}_0,\overline{e}_0\right][2,3]+\\ 3\left[\underline{e}_1,\overline{e}_1\right][2,3]\end{matrix}\right\}$	$\left[\underline{e}_0,\overline{e}_0\right][15,16]$	0

Equating the like coefficients of third row of Tables 2 and 3, it is obtained as given in equations (20) and (21).

$$\left\{\left[\underline{d}_0,\overline{d}_0\right][12,13]+3\left[\underline{d}_0,\overline{d}_0\right][2,3]\right\} = \left\{\begin{matrix}\left[\underline{e}_1,\overline{e}_1\right][15,16]+\left[\underline{e}_0,\overline{e}_0\right][2,3]+\\ 3\left[\underline{e}_1,\overline{e}_1\right][2,3]\end{matrix}\right\} \tag{20}$$

$$\left[\underline{d}_0,\overline{d}_0\right][10,11] = \left[\underline{e}_0,\overline{e}_0\right][15,16] \tag{21}$$

Using equations (15), (20) and (21), the values of $\left[\underline{d}_0,\overline{d}_0\right]$ and $\left[\underline{e}_1,\overline{e}_1\right]$ obtained are

$$\left[\underline{d}_0,\overline{d}_0\right] = [1.363,1.6], \quad \left[\underline{e}_1,\overline{e}_1\right] = [0.861,1.58] \tag{22}$$

Putting the values from equations (15) and (22), first-order model (14) turns out to be

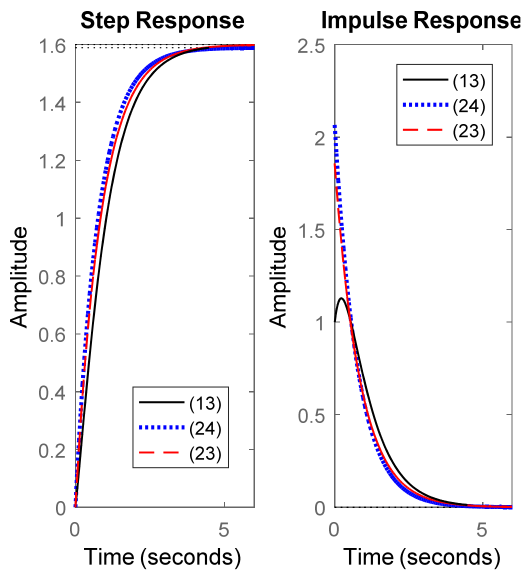
$$R(s) = \frac{[1.363,1.6]}{[1,1]+[0.861,1.58]s} \tag{23}$$

For second-order interval system given in equation (13), the model proposed by Bandyopadhyay et al. (1994) is given by equation (24).

$$R^B(s) = \frac{[12.58,19.072]}{[12,13]+[9.23,11.92]s} \tag{24}$$

Figure 1 depicts the step and impulse responses of second-order interval system (13) and models given in equations (23) and (24). It is evident from Figure 1 that step response of proposed model (23) is more closer to the step response of second-order interval system (13) than the step response of model (24) derived by Bandyopadhyay et al. (1994). The same is true for impulse response of proposed model. This shows that proposed multi-point Padé approximation provides excellent approximation in case of continuous SISO interval system.

Figure 1 Step and impulse responses of system and models (see online version for colours)



5 Proposed method for reduction of MIMO interval systems

The proposed method is also extended for the reduction of higher order MIMO continuous interval systems. The procedural steps are as follows.

Let the nth order MIMO interval system be

$$G(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) & \dots & G_{1j}(s) \\ G_{21}(s) & G_{22}(s) & \dots & G_{2j}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{i1}(s) & G_{i2}(s) & \dots & G_{ij}(s) \end{bmatrix} \tag{25}$$

with $i = 1,2,\dots,N$ and $j=1,2,\dots,M$ where M and N are, respectively, number of inputs and outputs. The transfer functions $G_{ij}(s)$ are given as

$$G_{ij}(s)=B_{ij}(s)/A_{ij}(s) \tag{26}$$

where $B_{ij}(s)$ and $A_{ij}(s)$ are, respectively, the numerator and denominator polynomials of ijth transfer function $G_{ij}(s)$.

Suppose, it is desired to obtain a kth order model as represented by

$$R(s)=\begin{bmatrix} R_{11}(s) & R_{12}(s) & \dots & R_{1j}(s) \\ R_{21}(s) & R_{22}(s) & \dots & R_{2j}(s) \\ \vdots & \vdots & \ddots & \vdots \\ R_{i1}(s) & R_{i2}(s) & \dots & R_{ij}(s) \end{bmatrix} \tag{27}$$

for equation (25) where $i = 1,2,\dots,N$ and $j = 1,2,\dots,M$, and

$$R_{ij}(s)=D_{ij}(s)/E_{ij}(s) \tag{28}$$

$D_{ij}(s)$ and $E_{ij}(s)$ are the numerator and denominator polynomials of ijth transfer function $R_{ij}(s)$ of MIMO model $R(s)$, respectively.

5.1 Determination of $R_{11}(s)$

The transfer function $R_{11}(s)$ is derived from $G_{11}(s)$. Suppose, $R_{11}(s)$ is given as

$$R_{11}(s)=\frac{\left[\underline{d_0},d_0\right]+\left[\underline{d_1},d_1\right]s+\left[\underline{d_2},d_2\right]s^2+\dots+\left[\underline{d_{k-1}},d_{k-1}\right]s^{k-1}}{\left[\underline{e_0},e_0\right]+\left[\underline{e_1},e_1\right]s+\left[\underline{e_2},e_2\right]s^2+\dots+\left[\underline{e_k},e_k\right]s^{k}}=\frac{D_{11}(s)}{E_{11}(s)} \tag{29}$$

where $\left[\underline{d_i},d_i\right]$ for $(i=0,1,\dots,k-1)$ and $\left[\underline{e_i},e_i\right]$ for $(i=0,1,\dots,k)$ are, respectively, interval coefficients of numerator and denominator polynomials provided $\left[\underline{e_0},e_0\right]=[1,1]$.

As assumed in case of SISO interval system in equation (11), the polynomial of $2k$ expansion points can be formed as

$$M_{11}(s)=\prod_{i=1}^{2k}(s-\lambda_i)=s^{2k}+\gamma_{2k-1}s^{2k-1}+\dots+\gamma_1 s+\gamma_0 \tag{30}$$

For equations (29), (5) and (6) modify to

$$P_{11}(s)=D_{11}(s)A_{11}(s) \tag{31}$$

$$Q_{11}(s)=B_{11}(s)E_{11}(s) \tag{32}$$

The interval coefficients of $D_{11}(s)$ and $E_{11}(s)$, are obtained by comparing the elements of Routh like arrays (Table 1) formed for $P_{11}(s)$ and $M_{11}(s)$, and $Q_{11}(s)$ and $M_{11}(s)$. In similar manner, other transfer functions $R_{12}(s)$, $R_{13}(s)$, …, $R_{ij}(s)$ can be determined.

6 Test case for MIMO system

Consider a second-order MIMO interval system (Sastry and Rao, 2003) as given below

$$G(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{33}$$

where

$$G_{11}(s)=\frac{B_{11}(s)}{A_{11}(s)}=\frac{[0.622,1.622]s+[1.00712,2.00712]}{[0.537464,1.537464]s^2+[1.379631,2.379813]s+[1,2]} \tag{34}$$

$$G_{12}(s)=\frac{B_{12}(s)}{A_{12}(s)}=\frac{[462.6,463.6]s+[715.2653,716.62653]}{[0.537464,1.537464]s^2+[1.379631,2.379813]s+[1,2]} \tag{35}$$

$$G_{21}(s)=\frac{B_{21}(s)}{A_{21}(s)}=\frac{[3.563,4.563]s+[4.8589,5.8589]}{[0.537464,1.537464]s^2+[1.379631,2.379813]s+[1,2]} \tag{36}$$

$$G_{22}(s)=\frac{B_{22}(s)}{A_{22}(s)}=\frac{[610.435,611.453]s+[1000.3485,1001.3485]}{[0.537464,1.537464]s^2+[1.379631,2.379813]s+[1,2]} \tag{37}$$

Suppose, a first-order MIMO model given by

$$R(s)=\begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix} \tag{38}$$

is desired for equation (33).

6.1 Determination of $R_{11}(s)$

The first-order transfer function $R_{11}(s)$ is given as

$$R_{11}(s)=\frac{D_{11}(s)}{E_{11}(s)}=\frac{\left[\underline{d_0},d_0\right]}{\left[\underline{e_0},e_0\right]+\left[\underline{e_1},e_1\right]s} \tag{39}$$

where

$$\left[\underline{e_0},e_0\right]=[1,1] \tag{40}$$

The $D_{11}(s)$ and $E_{11}(s)$ are obtained from $B_{11}(s)$ and $A_{11}(s)$ of matrix (33). For $D_{11}(s)$ and $E_{11}(s)$, expressions (31) and (32) become

$$\begin{aligned} P_{11}(s) &= D_{11}(s)A(s) \\ &= \left[\underline{d_0},d_0\right][0.5374,1.5374]s^2+\left[\underline{d_0},d_0\right][1.3791,2.379]s+\left[\underline{d_0},d_0\right][1,2] \end{aligned} \tag{41}$$

$$\begin{aligned} Q_{11}(s) &= B_{11}(s)E_{11}(s) \\ &= \left[\underline{e_0},e_0\right][0.622,1.622]s^2 \\ &\quad +\left\{\left[\underline{e_1},e_1\right][1.00721,2.00721]+\left[\underline{e_0},e_0\right][0.622,1.622]s\right\}+\left[\underline{e_0},e_0\right] \end{aligned} \tag{42}$$

The expansion points are considered as $s=0$ and $s=4$. For these expansion points, (30) becomes

$$M_{11}(s) = s^2 - 4s \tag{43}$$

The Routh like array for (41) and (43) is given in Table 4. Similarly, the Routh like array for equations (42) and (43) is provided in Table 5.

Table 4 Routh like array for $P_{11}(s)$ and $M(s)$

$\left[\underline{d_0}, d_0\right][0.5374, 1.5374]$	$\left[\underline{d_0}, d_0\right][1.379, 2.379]$	$\left[\underline{d_0}, d_0\right][1, 2]$
1	–4	0
$\left\{\left[\underline{d_0}, d_0\right][1.379, 2.379] + 4\left[\underline{d_0}, d_0\right][0.5374, 1.5374]\right\}$	$\left[\underline{d_0}, d_0\right][1, 2]$	0

Table 5 Routh like array for $Q_{11}(s)$ and $M(s)$

$\left[\underline{e_1}, e_1\right][0.622, 1.622]$	$\left\{\begin{matrix}\left[\underline{e_1}, e_1\right][1.00721, 2.00721] + \\ \left[\underline{e_0}, e_0\right][0.622, 1.622]\end{matrix}\right\}$	$\left[\underline{e_0}, e_0\right][1.00721, 2.00721]$
1	– 4	0
$\left\{\begin{matrix}\left[\underline{e_1}, e_1\right][1.00721, 2.00721] + \\ \left[\underline{e_0}, e_0\right][0.622, 1.622] + \\ 4\left[\underline{e_1}, e_1\right][0.622, 1.622]\end{matrix}\right\}$	$\left[\underline{e_0}, e_0\right][1.00721, 2.00721]$	0

By equating the like coefficients of third row of Tables 4 and 5, it is obtained as

$$\begin{aligned} \left[\underline{d_0}, d_0\right] &= [0.5036, 2.0072] \\ \left[\underline{e_1}, e_1\right] &= [0.0365, 4.727] \\ \left[\underline{e_0}, e_0\right] &= [1, 1] \end{aligned} \tag{44}$$

Putting the values from (44), the first-order transfer function (39) becomes

$$R_{11}(s) = \frac{D_{11}(s)}{E_{11}(s)} = \frac{[0.5036, 2.0072]}{[0.0365, 4.727]s + [1, 1]} \tag{45}$$

In similar manner, the transfer functions R_{12}, R_{21}, and R_{22} obtained are

$$R_{12}(s) = \frac{D_{12}(s)}{E_{12}(s)} = \frac{[358.1326, 716.62653]}{[0.3111, 2.2018]s + [1, 1]} \tag{46}$$

$$R_{21}(s) = \frac{D_{21}(s)}{E_{21}(s)} = \frac{[2.4294, 5.8589]}{[0.1330, 1.1712]s + [1, 1]} \tag{47}$$

$$R_{22}(s) = \frac{D_{22}(s)}{E_{22}(s)} = \frac{[500.2742, 1001.5484]}{[0.3347, 1.9035]s + [1, 1]} \tag{48}$$

The models proposed by Sastry and Rao (2003) for MIMO continuous interval system (33) are

$$R_{11}^{s}(s)=\frac{[0.2115,2.9]}{[1,1]s+[0.2,2.9]} \tag{49}$$

$$R_{12}^{s}(s)=\frac{[150.206,1038.58]}{[1,1]s+[0.2,2.9]} \tag{50}$$

$$R_{21}^{s}(s)=\frac{[1.0204,8.49]}{[1,1]s+[0.2,2.9]} \tag{51}$$

$$R_{22}^{s}(s)=\frac{[210.12,1452.24]}{[1,1]s+[0.2,2.9]} \tag{52}$$

The step and impulse responses of second-order interval transfer function (34), its proposed model (45) and model (49) as proposed by Sastry and Rao (2003) are plotted in Figure 2. From Figure 2, it is seen that the step response of proposed model (45) is nearer to that of the system (34) than that of the model given in equation (49). Also, it can be observed that the steady state of proposed model (45) is matching to that of the system (34) while it is deviating largely in case of equation (49). From the impulse responses as given in Figure 2, it is evident that the impulse response of proposed model (45) is closer to that of the system (34) when compared to the impulse response of equation (49). This proves that proposed model (45) is better approximant of the system (34) than model given in equation (49).

Figure 2 Step and impulse responses of system $G_{11}(s)$ and models $R_{11}(s)$ and R_{11}^{s} (see online version for colours)

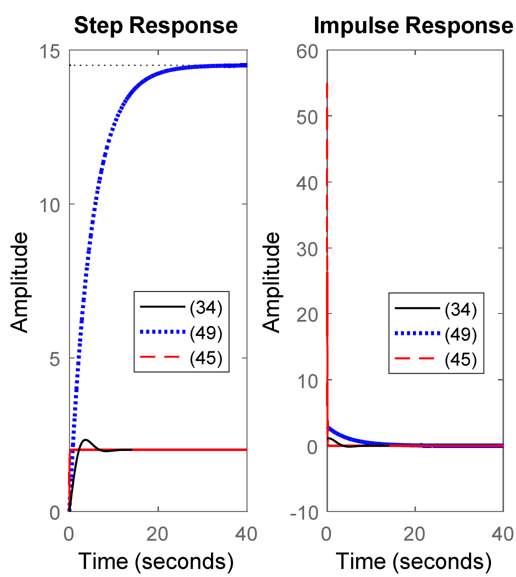
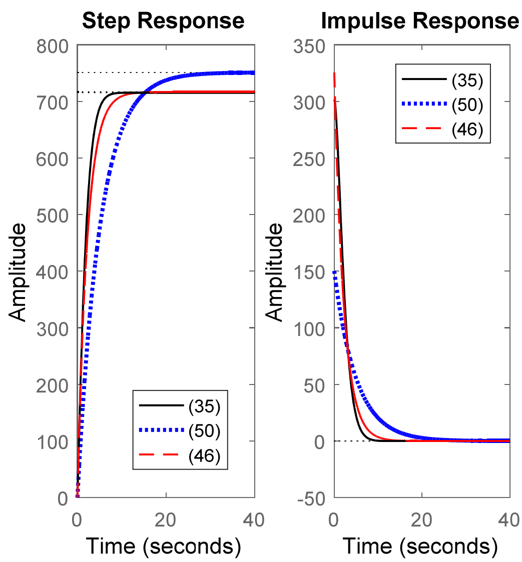


Figure 3 depicts the step and impulse responses of interval transfer function (35), its proposed model (46) and model (50) as proposed by Sastry and Rao (2003). It is clear from Figure 3 that the step response of proposed model (46) is in better proximity to that of the system (35) than that of model given in equation (50). It is also clear that the steady state of proposed model (46) is matched to that of the system (35) however is deviating for (50). The impulse responses provided in Figure 3 also show better matching of proposed model (46) to the system (35). So, it can be concluded that (46) is better approximant of equation (35) when compared to equation (50).

Figure 3 Step and impulse responses of system $G_{12}(s)$ and models $R_{12}(s)$ and R_{12}^{s} (see online version for colours)



The step response and impulse response of interval transfer function (36), its proposed model (47) and model (51) as proposed in equation (Sastry and Rao, 2003) are given in Figure 4. It can be seen from Figure 4 that the step response of proposed model (47) is closer to that of the system (36) than that of model given in equation (51). Also, it can be seen that the steady state of proposed model (47) is matched to the system (36) while is holding some deviation in case of equation (51). The impulse responses as provided in Figure 4 also depict better matching of proposed model (47) to the system (36). So, it can be said that (47) is better approximant than other.

Figure 5 plots the step and impulse responses of interval transfer function (37), its proposed model (48) and model (52) as derived in equation (Sastry and Rao, 2003). It is evident from Figure 5 that the step response of proposed model (48) is better matched to the step response of the system (37) than that of model given in equation (52). The same is true for impulse responses as plotted in Figure 5. So, it can be concluded that proposed model (48) is providing better approximation than (52).

Figure 4 Step and impulse responses of system $G_{21}(s)$ and models $R_{21}(s)$ and R_{21}^{s} (see online version for colours)

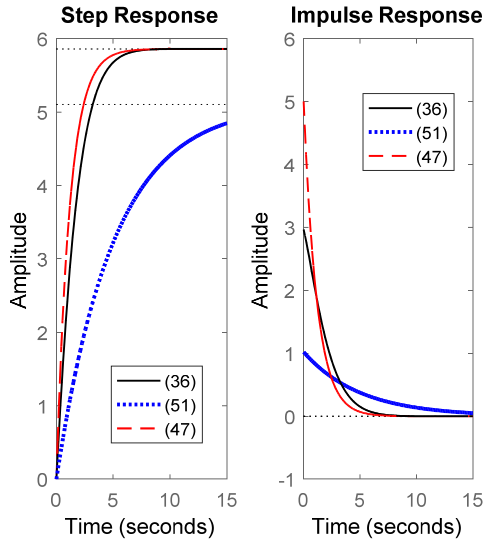
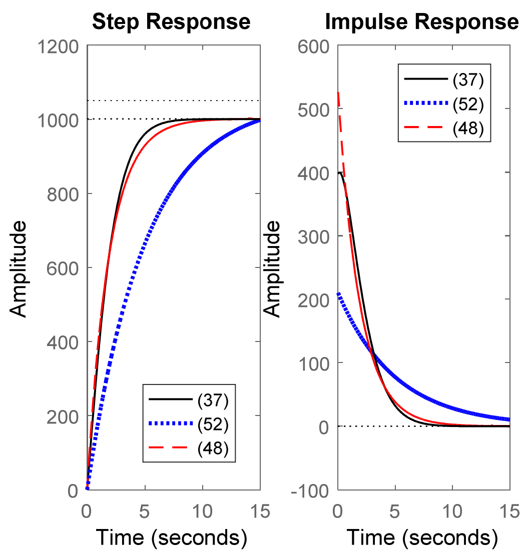


Figure 5 Step and impulse responses of system $G_{22}(s)$ and models $R_{22}(s)$ and R_{22}^{s} (see online version for colours)



7 Conclusion

In the present investigation, a multi-point Padé approximation is presented to reduce the order of continuous interval systems. First $2k$ expansion points are chosen around which the responses of higher order continuous interval system and its model are to be matched, where k is the order of model. It is observed in responses that approximation is improved due to matching around multi-points in case of multi-point Padé approximation. To show the efficacy of proposed method, first it is applied to a test case for SISO system. Then the same method is extended for reduction of MIMO interval system. A test case for reduction of MIMO interval system is also considered. From the results obtained for SISO case and MIMO case, it is confirmed that multi-point Padé approximation for continuous interval systems is producing better results. It is worth mentioning here that multi-point Padé approximation may produce even better results if combined with other reduction methods.

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