Controller design via model order reduction for interval systems using Kharitonov theorem and Nevalinna-pick theory: a case study

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Abstract: This manuscript describes a new procedure to design a controller for unstable high order interval systems using model order reduction and Nevalinna-pick theory. In this process, First, a high-order unstable interval system is converted into a reduced-order model in which improved Routh table truncation and time-moment matching techniques are applied. The denominator and numerator of reduced- order model are evaluated by improved Routh table truncation and time-moment matching technique, respectively. Next, employing the Kharitonov theorem and Nevalinna-pick theory, controller is designed for a reduced-order model in a certain range of uncertainty. To explain the procedure, the proposed method is applied to the fourth-order unstable interval system. Results prove the effectiveness of the method.

Keywords: controller design; interval system; Kharitonov theorem; model order reduction; Nevallina-pick theory; robust stabilisability; routh table truncation; Padé approximation.

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1 Introduction

During last decades, the control system design has been an important issue in the field of engineering. Several times, the controller design process becomes very complex due to uncertainty in the system which arises because of variation of system parameters, un-modelled dynamics, nonlinear dynamics of the plant, identification error, etc. It is quite difficult to take care of these uncertainties. To tackle these problems associated with uncertainty, robust controller is good choice.

Robust stability is covered in detail in literature. Popov criterion is generalised to problems with parametric uncertainty by Dahleh et al. (1993). Wei (1994) stabilised linear time-invariant plants via constant state feedback control. Shieh et al. (1995) digitally redesigned the cascaded analogue controller for the sampled-data interval system. The paper (De Santis and Vicino, 1996) utilised diagonal dominance for the robust stability of MIMO interval plants. Shieh et al. (1996) formulated a robust digital control law from a robust analogue control law for hybrid control of uncertain systems by converting continuous interval systems into discrete interval systems. Kimura (1984) designed a fixed controller for a class of transfer functions using the Nevanlinna-Pick theory. A few are also available on the robust stability Chapellat et al. (1993) of interval system.

Recently, several methods to design controllers for uncertain systems are proposed (Matuš and Prokop, 2016; Patre and Bhiwani, 2013; Kumar and Mummadi, 2020). In Matuš and Prokop (2016), authors proposed a technique to design a PID controller for interval systems. This technique is based on plotting the stability boundary locus in P-I plane. Patre and Bhiwani (2013) suggested a method of designing a robust controller for fuzzy parametric uncertain systems (FPUS). The method is based on switching of the FPUS into an interval state space controllable canonical form of system.

This manuscript is an extension of the Nevanlinna-Pick theory (Kimura, 1984) to the interval system using the Kharitonov theorem. It proposes an algorithm to reduce the order of unstable high order interval system into an unstable reduced order model (UROM) using Routh approximants and time-moment matching technique preceded by designing robust controllers to stabilise such UROM. As Kharitonov (1990) stated that every interval system could be written in the form of the Kharitonov rational systems. Keeping this in mind, Kharitononv theorem is applied to UROM to convert it into rational systems. Later, the Nevanlinna-Pick theory is utilised to design controllers for unstable systems around a certain uncertainty of unstable rational systems (Kimura, 1984).

The advantage of the proposed method is that it provides controller for unstable high order uncertain system in fixed range of uncertainty. The interval system is defined by one interval equation, but, controller designed is not expressed in terms of single interval equation. It is the limitation of the proposed method. Finally, an illustrative example is discussed which explains the method and its usefulness.

The remaining outline of the investigation is as follows: Section 2 describes Kharitonov theorem, Section 3 explains the procedure of model order reduction, the definition of robust stabilisability is given in Section 4, the Nevanlinna-Pick theory is discussed in Section 5, the condition for robust stabilisability is specified in Section 6, Section 7 justifies the method with an illustrative example, and finally conclusion is provided in Section 8.

2 Kharitonov theorem

Kharitonov (1990) suggested that an interval polynomial can be explained by its four vertex polynomials. Let, P_I be an interval polynomial where

$$P_I = [a_0^-, a_0^+] + [a_1^-, a_1^+]s + \cdots + [a_{n-1}^-, a_{n-1}^+]s^{n-1} + [a_n^-, a_n^+]s^n \tag{1}$$

Then, it's four vertex polynomials can be written as

$$\begin{aligned} p_1 &= a_0^- + a_1^- s + a_2^+ s^2 + \cdots \\ p_2 &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots \\ p_3 &= a_0^+ + a_1^- s + a_2^- s^2 + \cdots \\ p_4 &= a_0^+ + a_1^+ s + a_2^- s^2 + \cdots \end{aligned} \tag{2}$$

In this manner, each numerator and denominator of interval system has four vertex polynomials. Consider a proper interval system T_f given as

$$T_f = \frac{[n_0^-, n_0^+] + [n_1^-, n_1^+]s + \cdots}{[d_0^-, d_0^+] + [d_1^-, d_1^+]s + [d_2^-, d_2^+]s^2 + \cdots} \tag{3}$$

For equation (3), numerator vertex polynomials can be written as

$$\begin{aligned} n_1 &= n_0^- + n_1^- s + n_2^+ s^2 + \cdots \\ n_2 &= n_0^- + n_1^+ s + n_2^+ s^2 + \cdots \\ n_3 &= n_0^+ + n_1^- s + n_2^- s^2 + \cdots \\ n_4 &= n_0^+ + n_1^+ s + n_2^- s^2 + \cdots \end{aligned} \tag{4}$$

and in the same manner, denominator vertex polynomials for (3) are given as

$$\begin{aligned} d_1 &= d_0^- + d_1^- s + d_2^+ s^2 + \cdots \\ d_2 &= d_0^- + d_1^+ s + d_2^+ s^2 + \cdots \\ d_3 &= d_0^+ + d_1^- s + d_2^- s^2 + \cdots \\ d_4 &= d_0^+ + d_1^+ s + d_2^- s^2 + \cdots \end{aligned} \tag{5}$$

Total 16 rational transfer functions can be framed from equations (4) and (5) as presented by equation (6).

$$\begin{aligned} T_{11} &= \frac{n_1}{d_1}, T_{12} = \frac{n_1}{d_2}, T_{13} = \frac{n_1}{d_3}, T_{14} = \frac{n_1}{d_4} \\ T_{21} &= \frac{n_2}{d_1}, T_{22} = \frac{n_2}{d_2}, T_{23} = \frac{n_2}{d_3}, T_{24} = \frac{n_2}{d_4} \\ T_{31} &= \frac{n_3}{d_1}, T_{32} = \frac{n_3}{d_2}, T_{33} = \frac{n_3}{d_3}, T_{34} = \frac{n_3}{d_4} \\ T_{41} &= \frac{n_4}{d_1}, T_{42} = \frac{n_4}{d_2}, T_{43} = \frac{n_4}{d_3}, T_{44} = \frac{n_4}{d_4} \end{aligned} \tag{6}$$

3 Model order reduction

In the proposed method, initially high order unstable interval system is converted into a multiplication of two systems, i.e., stable high order system and unstable system. Later, stable high order system is converted into reduced order model (ROM) using the techniques proposed in Dolgin (2005) and Singh et al. (2017). In this method, the denominator and numerator of ROM are derived by Dolgin (2005) and Singh et al. (2017), respectively. The reason behind doing this is that the use of Dolgin (2005) produces stable ROM for stable system. Simultaneously, derived formulation for time moments in Singh et al. (2017) confirms better steady-state response matching. The outline of the method is given as follows.

Let high order unstable interval system be

$$T_{UF} = \frac{[n_0^-, n_0^+] + [n_1^-, n_1^+]s + \cdots + [n_n^- - 1, n_n^+ - 1]s^n - 1}{[d_0^-, d_0^+] + [d_1^-, d_1^+]s + [d_2^-, d_2^+]s^2 + \cdots + [d_n^-, n_n^+]s^n} \tag{7}$$

It can also be re-written (Appendix A) as

$$T_{UF} = \frac{\sum_{i=0}^{y-1}[n_{i_u}^-, n_{i_u}^+]s^i}{\sum_{i=0}^{y}[d_{i_u}^-, d_{i_u}^+]s^i} \times \frac{\sum_{i=0}^{x-1}[n_{i_s}^-, n_{i_s}^+]s^i}{\sum_{i=0}^{x}[d^- i_s, d_{i_s}^+]s^i} \tag{8}$$

$$T_{UF} = T_{UF_1} T_{SHF} \tag{9}$$

where T_{UF_1} is unstable interval system and T_{SHF} is high order stable transfer function. From (8), T_{SHF} is given as

$$T_{SHF} = \frac{\sum_{i=0}^{x-1}[n_{i_s}^-, n_{i_s}^+]s^i}{\sum_{i=0}^{x}[d^- i_s, d_{i_s}^+]s^i}. \tag{10}$$

For the sake of calculations, equation (10) can also be denoted as

$$T_{SHF} = \frac{n_0 + n_1 s + \cdots + n_{y-1}s^{y-1}}{d_0 + d_1 s + \cdots + d_y s^y} \tag{11}$$

T_{SHF} is converted into T_{SRM}, i.e., stable ROM. In this work, firstly, the denominator of ROM is obtained from the method proposed in Dolgin (2005) and later, numerator of ROM is calculated by the time moment matching technique proposed in Singh et al. (2017). The detailed procedure for obtaining the denominator and numerator is discussed as follows.

3.1 Denominator

Let the denominator of high order interval system (11) be

$$D_{T_{SHF}} = [d_{0_s}^-, d_{0_s}^+] + [d_{1_s}^-, d_{1_s}^+]s + ... + [d_{x_s}^-, d_{x_s}^+]s^x \tag{12}$$

Routh table for (12) is given as

Table 1 Routh table

s^x	$[d_{x_s}^-, d_{x_s}^+]$	$[d_{(x-2)_s}^-, d_{(x-2)_s}^+]$	$\cdots$	$\cdots$
s^{x-1}	$[d_{(x-1)_s}^-, d_{(x-1)_s}^+]$	$[d_{(x-3)_s}^-, d_{(x-3)_s}^+]$	$\cdots$	
$\vdots$	$\vdots$	$\ddots$		
s	$[d_{(x,1)_s}^-, d_{(x,1)_s}^+]$			
s^0	$[d_{(x+1,1)_s}^-, d_{(x+1,1)_s}^+]$			

where

$$d_{(i,j)_s} = d_{(i-2,j+1)_s} - \frac{\hat{d}_{(i-2,1)}}{\hat{d}_{(i-1,1)}} \times d_{(i-2,j+1)_s} \tag{13}$$

In equation (13), the interval subtraction operation Dolgin (2005) is taken as

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_1, a_2 - b_2] \tag{14}$$

To ensure the existence of $\hat{d}_{(i,j)_s}$ Dolgin (2005), the uncertainty in $d_{(i-1,j+1)_s}$ is narrowed in the following way:

$$d_{(i-1,j+1)_s} = \Big[max(\underline{d}_{(i-1,j+1)_s}, \hat{d}_{(i-1,j+1)_s} - \frac{1}{2}K.L_{(i-2,j+1)}) \\ min(\bar{d}_{(i-1,j+1)_s}, \hat{d}_{(i-1,j+1)_s} + \frac{1}{2}K.L_{(i-2,j+1)})\Big] \tag{15}$$

where $L_{(i-2,j+1)} = \bar{d}_{(i-2,j+1)_s} - \underline{d}_{(i-2,j+1)_s}$, $\bar{d}_{(i-2,j+1)_s}$ and $\bar{d}_{(i-2,j+1)_s}$ are upper limit and lower limit of interval coefficient $\bar{d}_{(i-2,j+1)_s}$, $\hat{d}_{(i,j)_s}$ is the mid-point of the interval $\hat{d}_{(i,j)_s}$, and $K = \frac{|\hat{d}_{(i-1,1)_s}|}{|\hat{d}_{(i-1,2)_s}|+|\hat{d}_{(i-1,1)_s}|}$.

Using Table 1, denominator of ROM can be written as

$$D_{T_{SRM}} = [d_{(x-r+1,1)_s}^-, d_{(x-r+1,1)_s}^+]s^r + [d_{(x-r+2,1)_s}^-, d_{(x-r+2,1)_s}^+]s^r + \cdots \tag{16}$$

For the sake of calculation, the denominator of ROM is denoted as

$$D_{T_{SRM}} = \hat{d}_r s^r + \hat{d}_{r-1} s^{r-1} + ... + \hat{d}_1 s + d_0 \tag{17}$$

3.2 Numerator

After evaluating denominator, numerator of ROM can be calculated as follows.

Suppose

$$N_{T_{SRM}} = \hat{n}_r s^r + \hat{n}_{r-1} s^{r-1} + ... + \hat{n}_1 s + \hat{n}_0 \tag{18}$$

Using equations (17) and (18), ROM can be written as follows

$$T_{SRM} = \frac{N_{SRM}}{D_{SRM}} \tag{19}$$

In terms of time-moments, equations (11) and (19) can also be given as

$$T_{SHF} = t_0 + t_1 s + t_2 s^2 + ... \tag{20}$$

$$T_{SHM} = \hat{t}_0 + \hat{t}_1 s + \hat{t}_2 s^2 + ... \tag{21}$$

where

$$\hat{t}_k = \frac{\hat{n}_k}{\hat{d}_0} + \sum_{i=0}^{k-1} \frac{\hat{d}_{k-i}\hat{t}_i}{\hat{d}_0}, k = 0, 1, 2, ... \tag{22}$$

and

$$t_k = \frac{n_k}{d_0} + \sum_{i=0}^{k-1} \frac{d_{k-i} t_i}{d_0}, k = 0, 1, 2, ... \tag{23}$$

On comparing time-moments as

$$\hat{t}_k = t_k, k = 0, 1, 2, ... \tag{24}$$

numerator coefficients of ROM can be calculated.

4 Robust stabilisability

Robust stabilisability Kimura (1984) is a very important condition for interval in class. Following definitions are used for robust stabilisability.

Definition 1: A class $C(f_0(s), v(s))$ is defined for a rational transfer function $f(s)$ if

- $f(s)$ and $f_0(s)$ must have same number of poles on right hand side of $s-$plain (unstable poles).
- $|f(s) - f_0(s)| \leq |v(s)|, |v(s)| > 0, \forall s$

where, $s = jw$ and w is a real number.

In this definition, $f_0(s)$ is nominal transfer function and $v(s)$ is uncertainty in the form of proper transfer function.

Definition 2: As shown in Figure 1, a closed loop system with negative feedback is stable iff for each $f(s)$ in class $C(f_0(s), v(s))$, there exists a controller $c(s)$. An inequality is proposed in Kimura (1984), for achievement of this purpose.

$$|v(jw)c(jw)| < |1 + f_0(jw)c(jw)|, \forall w \tag{25}$$

Later using equation (25), equality is defined as

$$1 + f_0(jw)c(jw) = \frac{c(jw)}{\mu(jw)}. \tag{26}$$

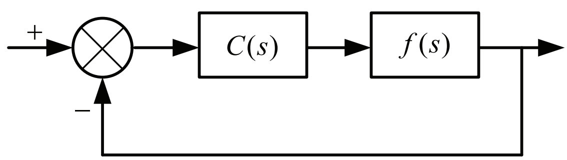
Hence

$$\mu(s) = \frac{c(s)}{1 + f_0(s)c(s)} \tag{27}$$

and controller $c(s)$ can be written as

$$c(s) = \frac{\mu(s)}{1 - f_0(s)\mu(s)} \tag{28}$$

Figure 1 Controller with plant



5 The Nevalinna-Pick theory

In complex analysis, interpolation of data utilises holomorphic function (Ball and Trent, 1998). Later is obtained by Nevanlinna-Pick theory (NPT) (Appendix B). It is also used in circuit theory Delsarte et al. (1981), signal processing Dewilde et al. (1978) and approximation problems Kimura (1983).

Definition 3: Consider an analytic function $\phi(s)$ with $Re(s) \geq 0$ satisfying the inequality

$$|\phi(jw)| \leq 1, \forall w \tag{29}$$

The inequality (29) is bounded real (BR). But if

$$|\phi(jw)| < 1, \forall w \tag{30}$$

then it is called strongly bounded real (SBR). Now, interpolation problem is formulated for SBR or BR functions. These are called Nevanlinna-Pick problems.

Let us say, there are $2n$ complex numbers χ_i, δ_i, $i = 1, 2, ..., n$, are following the inequality as given by

$$Re[\chi_i] > 0, |\delta_i| < 1, i = 1, 2, ..., n \tag{31}$$

NPT is used to determine $\phi(s)$, which fulfills the following relation

$$\phi(\chi_i) = \delta_i, i = 1, 2, .., n \tag{32}$$

It is accomplished by forming Fenyves array $\delta_{i,j}$ as given by

$$\begin{aligned} &\delta_{i,1} = \delta_i, i = 1, 2, ..., n \\ &\delta_{i,j+1} = \frac{(\chi_i + \bar{\chi}_j)(\delta_{i,j} - \delta_{j,j})}{(\chi_i - \chi_j)(1 - \delta_{i,j}\bar{\delta}_{j,j})} 1 \leq j \leq i - 1 \leq n - 1 \\ &\rho_j = \delta_{j,j} \end{aligned} \tag{33}$$

Thus, $\phi(s)$ is solved by iterating linear fractional transformations as

$$\phi_j(s) = \frac{(s - \chi_j)\phi_{j+1}(s) + \rho_j(s + \bar{\chi}_j)}{s + \bar{\chi}_j + \bar{\rho}_j(s - \chi_j)\phi_{j+1}(s)}, j = n, n-1, ..., 1, \tag{34}$$

$$\phi(s) = \phi_1(s) \tag{35}$$

In (34), $\phi_{n+1}(s)$ is SBR function. Initialisation and termination conditions are given as

$$\begin{aligned} &\phi(0) = \delta_0 \\ &\phi(\infty) = \delta_{n+1} \end{aligned} \tag{36}$$

where

$$|\delta_0| < 1, |\delta_{n+1}| < 1 \tag{37}$$

6 The condition of robust stabilisability

Let, f_0 be an n^{th} order nominal transfer function and it contains $\chi_1, \chi_2, ..., \chi_k$ where k is the number of unstable poles. Blaschke product Kimura (1984) for these unstable poles can be written as follows

$$B(s) = \frac{(\chi_1 - s)...(\chi_k - s)}{(\bar{\chi}_1 + s)...(\bar{\chi}_k + s)} \tag{38}$$

It will satisfy the condition as written by

$$|B(jw)| = 1, \forall w \tag{39}$$

Suppose

$$\tilde{f}_0(s) = f_0 B(s)$$
$$\delta_i = \frac{v_m(\chi_i)}{\tilde{f}(\chi_i)} \tag{40}$$

Applying equations (34), (35), (38) and (40), $\mu(s)$ can be formulated as follows

$$\mu(s) = \frac{B(s)}{vm(s)}\phi(s) \tag{41}$$

where $v_m(s)$ is an arbitrary SBR function. Controller $c(s)$ is designed using equations (28) and (41).

7 Example

Let an unstable interval transfer function,

$$\begin{aligned} T_{UF} &= \frac{[2,3]s^3 + [19.5, 21.5]s^2 + [32.5, 34.5]s + [15, 16]}{[2,3]s^4 + [15.7, 22.2]s^3 + [26.9, 61.2]s^2 + [4.3, 71.9]s + [-9.7, 30.1]} \\ &= \frac{s + [1,1]}{s + [-0.45, 1.4]} \times \frac{[2,3]s^2 + [317.5, 18.5]s + [15, 16]}{[2,3]s^3 + [17, 18]s^2 + [35, 36]s + [20.5, 21.5]} \\ &= T_{UF_1} \times T_{SHF} \end{aligned} \tag{42}$$

where T_{UF_1} stands for unstable interval transfer function and T_{SHF} stands for stable high order interval transfer function.

In proposed technique, T_{SHF}, i.e., high order system is converted into stable ROM using technique proposed in Singh et al. (2017). Then, controller is designed for the product of T_{UF_1} and T_{SHF}.

Using equations (8)–(24), T_{SRM}, i.e., ROM of T_{SHF} can be written as

$$T_{SRM} = \frac{[0.44, 0.52]}{s + [0.62, 0.67]} \tag{43}$$

So, from equations (42) and (43), it is obtained

$$\begin{aligned} T_{URM} &= T_{UF1} \times T_{SRM} \\ &= \frac{s + [1,1]}{s + [-0.45, 1.4]} \times \frac{[0.44, 0.52]}{s + [0.62, 0.67]} \end{aligned} \tag{44}$$

Hence

$$T_{URM} = \frac{([0.44, 0.52])(s+1)}{s^2 + [0.17, 2.07]s + [-0.30, 0.94]} \tag{45}$$

Now, Kharitonov polynomials for T_{URM} can be written as

$$f_{0_1} = \frac{[0.44, 0.52](s+1)}{(s+0.64)(s-0.47)} \tag{46}$$

$$f_{0_2} = \frac{[0.44, 0.52](s+1)}{(s+2.21)(s-0.14)} \tag{47}$$

$$f_{0_3} = \frac{[0.44, 0.52](s+1)}{(s+1.40)(s+0.67)} \tag{48}$$

and

$$f_{0_2} = \frac{[0.44, 0.52](s+1)}{(s+0.09-0.97i)(s+0.09+0.97i)} \tag{49}$$

Using the proposed algorithm, controllers are designed for unstable systems around a certain uncertainty of first two unstable rational systems (i.e., f_{0_1} and f_{0_2}). Remaining systems (i.e., f_{0_3} and f_{0_4}) are stable. So, there is no need to design controller for such systems. Leaving the interval coefficient, the step wise controller designing procedure for f_{0_1} is given as

As provided in equation (46), suppose

$$\begin{aligned} b_1 &= 1 \\ \chi &= 0.47 \\ b_2 &= 0.64 \end{aligned} \tag{50}$$

Using

$$f_0 = \frac{(s+b_1)}{(s-\chi)*(s+b_2)} \tag{51}$$

Applying equation (38), it is obtained

$$\begin{aligned} B &= \frac{\chi - s}{\chi + s} \\ &= \frac{0.47 - s}{0.47 + s} \end{aligned} \tag{52}$$

Using equations (52), (40) is given

$$\tilde{f}_0 = B * f_0 \tag{53}$$

Let, the SBR function, v_m, is given by

$$v_m = \frac{b}{(s+1)} \tag{54}$$

and applying equation (40), it is obtained

$$\delta_1^{0.47} = \frac{f_0^{0.47}}{v_m^{0.47}}$$
$$\delta_1^{0.47} = \frac{-(b(s+\frac{16}{25})(s+\frac{47}{100}))}{(s+1)^2} \tag{55}$$

Putting $\chi = 0.47$ in equation (55), it is obtained

$$\delta_1^{0.47} \approx -0.48b$$

Using the inequality of equation (37), it can be written as

$$|\delta_1^{0.47}| < 1 \tag{56}$$

Hence

$$|b| < 2.07 \tag{57}$$

and from equation (56), it is obtained

$$k = 2.07 \tag{58}$$

Using equations (33), (34), (35) and (41), it is obtained

$$p_1 = \frac{b}{k}$$
$$\phi_2 = \frac{b * s}{k * s + k}$$
$$\phi_1 = \frac{((s - b_1) * \phi_2 + p_1 * (s + b_2))}{((s + b_2) + p_1 * (s - b_2) * \phi_2)}$$
$$\mu_1 = B * \frac{\phi_1}{v_m} \tag{59}$$

Finally, using equation (28), controller for the class $C(f_{0_1}, v_m)$ of unstable transfer functions is given by

$$c_1 = \frac{\mu_1}{1 - f_{0_1} * \mu_1} \tag{60}$$

$$c_1 = \frac{-125.26s^5 - 209.18s^4 - 86.3s^3 + 11.43s^2 + 22.68s + 8.85}{[(30.24b^2 + 254.96)s^4 + (19.35b^2 + 647.79)s^3 + (591.58 - 66.80b^2)s^2 + (246.52 - 4.27b^2)s + 477.74]}, \tag{61}$$

where $0 < b < 2.07$.

Applying same procedure as above, the controller for the class $C(f_{0_2}, v_m)$ is given as

$$c_2 = \frac{(-267.23s^5 - 954.01s^4 - -899.39s^3 - 209.99s^2 + 8.41s + 5.78)}{[(67.57b^2 + 531.13)s^4 + (159.51b^2 + 1589.09)s^3 + (1378.75 - 1.33b^2)s^2 + (350.93 - 2.93b^2)s + 30.14]}, \quad (62)$$

where $0 < b < 1.98$.

After calculating the range of b for each unstable nominal transfer function of equation (45), a random value of b from each range is selected. Corresponding to these values, unstable transfer function and controller for each class is derived, after that using these two of the same class, closed loop transfer function for each class is calculated, as given in Table 2.

Table 2 Information regarding step and impulse responses

S. no.	*Unstable transfer-function* $f = f_{0_i} + \frac{b}{s+1}$;	*Corresponding controller*	*Range of b*	*Random value of b*	*Step and impulse responses of corresponding closed loop system i.e.* $\frac{cf}{1+cf}$ *in Figure 2*
1.	$f = f_{0_1} + \frac{b}{s+1}$	c_1	$0 < b < 2.07$	1.1	F11 and F12
2.	$f = f_{0_2} + \frac{b}{s+1}$	c_1	$0 < b < 1.98$	1.2	F21 and F22

Figure 2 Controller with plant (see online version for colours)

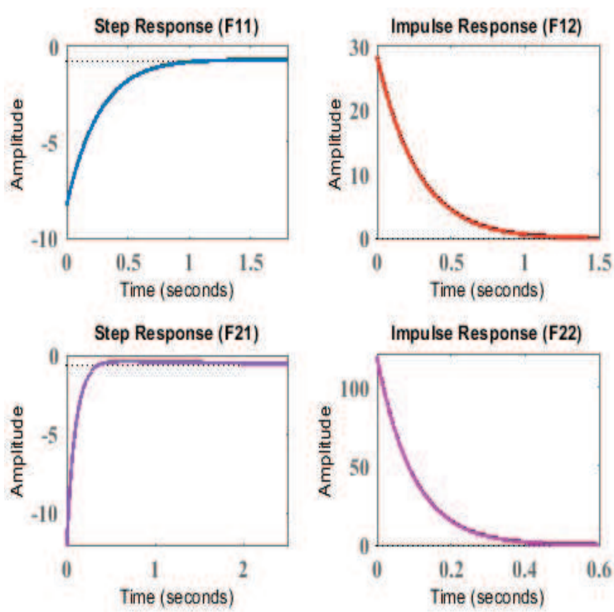
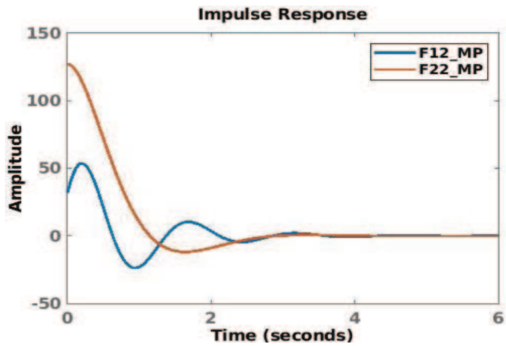


Figure 3 Impulse response of unstable system using technique proposed by Matuš and Prokop (2016) (see online version for colours)



Further, step responses (i.e., F11 and F21) and impulse responses (i.e., F12 and F22) of these closed loop transfer functions are plotted in Figure 2. It is clear from this figure that step response of each system reaches to a steady state value and impulse response diminishes to zero which is necessary and sufficient condition for stability, i.e., bounded output for bounded input (BIBO).

Simultaneously, impulse responses of the systems (i.e., F12 and F22) with respective controllers proposed by Matuš and Prokop (2016) are provided in Figure 3. It is clear from Figures 2 and 3 that time taken by the output to reach steady state value by the proposed technique is less than Matuš and Prokop (2016). Also, the impulse responses given in Figure 3 contain overshoots which are absent in Figure 2. It shows effectiveness of the controller designed by the proposed method over Matuš and Prokop (2016).

8 Conclusion

In this article, a process of controller design by model order reduction of interval system is discussed. In this process, high order interval system is converted into ROM using Routh approximants and time moment matching technique followed by controller design for ROM using Kharitonov theorem and Nevalinna-Pick theory. Kharitonov theorem is applied to convert interval system into equivalent rational systems. Controllers are designed around these rational systems in a certain range of uncertainty using Nevanlinna-Pick theory. It shows that Nevanlinna-Pick theory can also be used for robust stability. Robust stability and condition for robust stability of uncertain system are discussed. At the last, an illustrative example of fourth order unstable interval system is discussed. Results prove the usefulness of applied method.

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Appendix A

Consider an nth order interval system:

$$T_f = \frac{[n_{n-1}^-, n_{n-1}^+]s^{n-1} + [n_{n-2}^-, n_{n-2}^+]s^{n-2} + \cdots + [n_0^-, n_0^+]}{[d_n^-, d_n^+]s^n + [d_{n-1}^-, d_{n-1}^+]s^{n-1} + \cdots + [d_0^-, d_0^+]} \tag{A.1}$$

The denominator of T_f is given by

$$D_n = [d_n^-, d_n^+]s^n + [d_{n-1}^-, d_{n-1}^+]s^{n-1} + \cdots + [d_0^-, d_0^+] \tag{A.2}$$

Let, factorization of (64) is given by

$$D_n = (s + [\alpha_0^-, \alpha_0^+])([\beta_{n-1}^-, \beta_{n-1}^+]s^n + [\beta_{n-2}^-, \beta_{n-2}^+]s^{n-2} + \cdot + [\beta_0^-, [\beta_0^+]) \tag{A.3}$$

If we consider the coefficient of s^0 from equations (64) and (65), then

$$[\alpha_0^-, \alpha_0^+] \times [\beta_0^-, \beta_0^+] = [d_0^-, d_0^+] \tag{A.4}$$

In equation (66), there are four unknowns (i.e., α_0^-, α_0^+, β_0^- and β_0^+) and two equations as given by

$$\begin{aligned} \alpha_0^- \beta_0^- &= d_0^- \\ \alpha_0^+ \beta_0^+ &= d_0^+ \end{aligned} \tag{A.5}$$

So, there will be many solutions of (67). From above analysis, it is clear that factorization of an interval polynomial is not unique.

Appendix B

The Nevanlinna- Pick theorem given as: Let, $w_1, w_2 \cdots, w_n, z_1, z_2, \ldots, z_n \in D$. There exists a holomorphic function $f : D \to D$ such that $f(w_i) = z_i$ (where D is unit disc in complex plane whose centre is at origin), iff the Pick matrix

$$\left(\frac{1 - \bar{z}_i z_j}{1 - \bar{w}_i w_j}\right)_{i,j=1}^n$$

is positive semi-definite.