Design optimisation of circular piezoelectric bimorph actuators using finite element analysis

C. Rakeshkumar*

Department of Aeronautical Engineering,
Vel-Tech Rangarajan Dr. Sagunthala R&D Institute
of Science and Technology,
Tamil Nadu, Chennai-600062, India
Email: rakeshae0843@gmail.com
*Corresponding author

R. Naveen

Department of Aeronautical Engineering,
Bannari Amman Institute of Technology,
Sathyamangalam, Tamil Nadu, Chennai-638401, India
Email: raga.aero94@gmail.com

M. Saleem and R. Raga

Department of Aeronautical Engineering,
Vel-Tech Rangarajan Dr. Sagunthala R&D Institute
of Science and Technology,
Tamil Nadu, Chennai-600062, India
Email: strokesaleem1@gmail.com
Email: aeronaveen@gmail.com

Abstract: In this work, finite element simulation is used to optimise a circular piezoelectric bimorph bender actuator. The piezoelectric bender actuator consists of a circular base membrane with a circular piezoelectric bimorph bonded to one side of the base membrane by epoxy glue. Many parameters have been considered during preliminary design, in which the oscillating membrane thickness, lead-zirconate titanate (PZT) thickness, input voltage and frequency play a vital role. The piezoelectric plate will oscillate the membrane during alternating voltage is applied towards the piezoelectric plate. Once the pulse voltage is applied to the piezoelectric plate, convex/concave deformation will occur on the structure. To obtain extreme deflection of the actuator, it is imperative that thickness ratio of bimorph to the base plate is optimised. Keeping the base plate diameter and the thickness constant, a parametric analysis is performed to find the bimorph diameter and its thickness for maximum deflection. The piezoelectric sheet thickness is chosen by iterating various thickness of the piezoelectric plate to actuate the membrane. The numerical simulation results show that for certain Aluminium membrane thickness, there will be an optimal coupling thickness of the piezoelectric plate. Transient analysis of this membrane is obtained by changing the voltage (200V, 0V, –200V, 0V) alternatively in equal intervals of time.

Keywords: piezo electric material; FEA; bimorph actuator; deflection; ANSYS.

Reference to this paper should be made as follows: Rakeshkumar, C., Naveen, R., Saleem, M. and Raga, R. (2021) 'Design optimisation of circular piezoelectric bimorph actuators using finite element analysis', *Int. J. Computational Materials Science and Surface Engineering*, Vol. 10, No. 1, pp.27–42.

Biographical notes: C. Rakeshkumar is working as an Assistant Professor of the Aeronautical department at Vel-Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology. He received his M.E from Anna University, Tirunelveli (2014). He has seven years of teaching experience in the aeronautical domain. His area of interest is a characterisation of composite materials and micro aerial vehicle.

R. Naveen is working as an Assistant Professor of the Aeronautical Department at Bannari Amman Institute of Technology. He received his BE from Vel Tech Engineering College (2012), Chennai and ME from Anna University, Tirunelveli (2015). He is currently pursuing his Doctorate in the area of composite materials at Anna University, Chennai. He has six years of teaching experience in the aeronautical domain. His area of interest is a characterisation of composite materials and conceptual design of flying car concepts. He has professional life member in ISTE from 2017 onwards.

M. Saleem is working as an Assistant Professor of the Aeronautical Department at Vel-Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology. He received his ME from Madras Institute of Technology (2015). He has five years of teaching experience in the aeronautical domain. His area of interest is a characterisation of composite materials and unmanned aerial vehicle.

R. Raga is working as an Assistant Professor of the Aeronautical Department at Vel-Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology. He received his ME from Anna University, Tirunelveli (2017). He has three years of teaching experience in the aeronautical domain. His area of interest is a characterisation of composite materials.

RETRACTED

1 Introduction

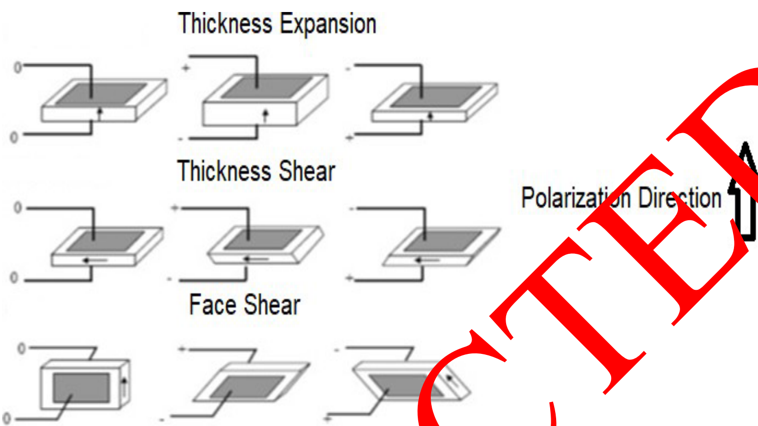
1.1 Piezoelectric effect

It is electrically polarised by the rare characteristic of induced crystalline, when it is applied to a mechanical force. The compressive and tensional force would therefore produce voltages of opposite polarity, which will be in some relation to the force applied (Murugu Nachippan et al., 2015). After that the converse connection needs to be confirmed, then it attains the polarity of the field in extended or shortened form by the voltage creating crystal and it needs to be exposed to an electric field. Such characteristics were graded according to the piezoelectric effect and the inverse piezoelectric effect. Though the scales of the piezoelectric voltages, travel direction or force are tiny and it frequently requires amplification, it refers a certain plate of piezoelectric ceramic may get surge or reduce in thickness by small portion of a millimetre (Mathew Alphonse and Ramesh Kumar, 2017).

1.2 Basic piezoelectric modes

Figure 1 explains the basic piezoelectric modes

Figure 1 Basic piezoelectric modes



1 *Thickness expansion*

In this mode, when the voltage is applied along the direction of polarisation, the piezoelectric material will be expanded or contracted. When the Polarisation direction is towards the Positive terminal then expansion will occur and if Polarisation Direction is opposite to the positive terminal then contraction will occur (Murugu Nachippan et al., 2015).

2 *Thickness shear*

In this mode, when the voltage is applied upright to the direction of polarisation, the piezoelectric material is then sheared. When the voltage is applied perpendicular to the thickness, Shear thickness may occur (Smits and Ballato, 1994).

3 *Face shear*

Face shear is similar to thickness shear but the orientation is the only modification, which may change the polarisation direction and load applied direction.

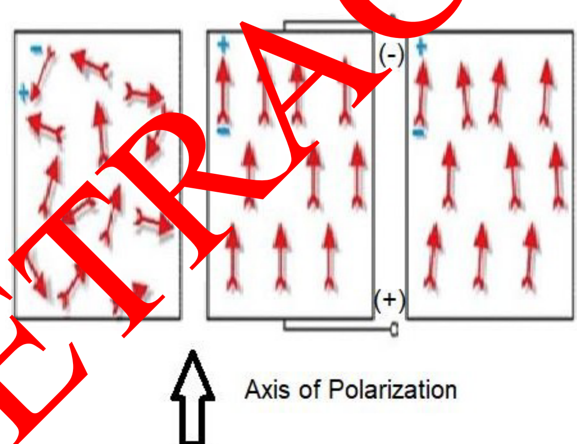
1.3 Poling of the piezoelectric ceramics

In recent years, piezoelectric and inverse piezoelectric effect is used in many new applications by employing the techniques like ceramic which is made by metal oxide to design. So, its physical properties become strong, it is chemically inert and it is cost effective to manufacture, when compare to the current techniques (DeVoe and Pisano, 1997). In other purposes, the form, dimension and internal composition of ceramic, i.e., piezoelectric ceramics may be rendered to order. Approximately the ceramics can be produced from the combinations of lead, zirconate, titanate which display the better sensitivity and it can have better operating temperature when compare to the other ceramic compositions which is most widely used (Mu et al., 1999). It is not possible to change the piezoelectric ceramic into piezoelectric material, without the align the random ferroelectric domain and it could be finished by the process called as poling. This poling influences the D.C voltage throughout the material. The alignment of ferroelectric domain leads the induced field in piezoelectric effect, where alignment should be monitored because the ferroelectric domain should not align completely and it has to partially align.

The poling voltage, voltage timing detained on the material and temperature is totally depends on the alignment of domain (He et al., 2000). When poling the material will be eternally increase in measurement between the electrodes (i.e.) poling electrode and decrease when it is parallel to the electrode.

De-poling the material is possible by moving back the poling voltage or attain curie point stage by drastically increasing the temperature of the material and its possible to change by applying mechanical stress induced on the material. Ceramic elements domain can be aligning when it is subjected to electric field, basically it is considered to be point below the temperature of curie point (Figure 2-middle).

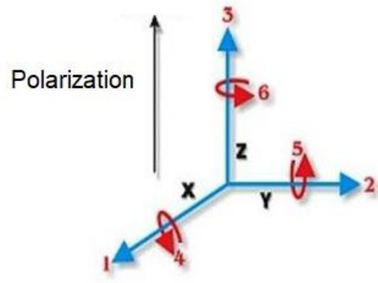
Figure 2 Polarising (poling) a piezoelectric ceramic, random orientation of polar domains prior to polarisation (left), polarisation in DC electric field (middle), remanent polarisation after electric field is removed (right) (see online version for colours)



1.4 Directions of forces affecting the piezoelectric element

Since a piezoelectric ceramic is anisotropic, both the path of the electric force or mechanical applied and the directions upright to the force applied are linked to the physical constants. Therefore, each constant typically has two subscriptions showing the directions of the two associated quantities, such as stress and elasticity pressure (Bisegna and Caruso, 2001).Typically the path of positive polarisation matches with the third axis of a rectangular configuration of the *X*, *Y*, and *Z* axes (Figure 3). Direction *X*, *Y*, or *Z* is represented respectively by subscription 1, 2, or 3, and shear along one of these axes is represented respectively by subscription 4, 5, or 6.

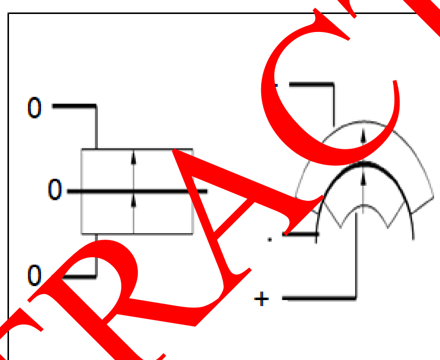
Figure 3 Directions of forces affecting the piezoelectric element (see online version for colours)



1 *Piezoelectric benders*

Frequently the piezoelectric benders are used to generate the actuators with huge displacement abilities. The benders will work as same as the bimetallic spring works. (9) Figure 4 shows the typical view of the piezoelectric bender.

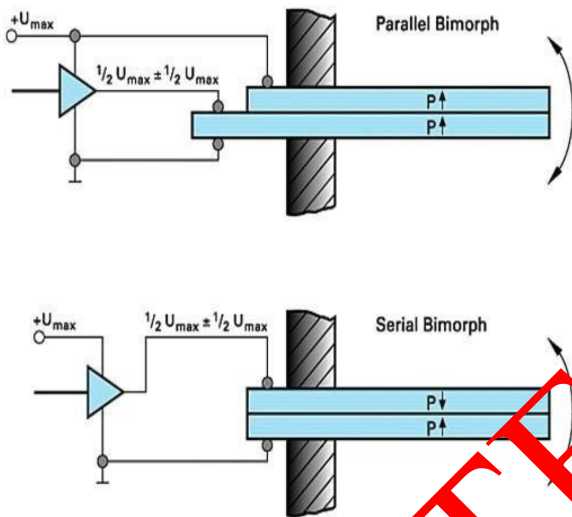
Figure 4 Piezoelectric bender



1.5 PZT bimorph actuator configuration

Micro pump drivers, displacement actuators and the sound generators or receiving devices are replaced by piezoelectric bimorphs. The modest bimorph is fabricated with the help of bonding the Piezoelectric element of a passive element bye side by side. By applying the electric current to the piezo electric strains by thickness side in the form of transversely and radially. The entire biomorphic structure will be bend due to radial strains which cases contraction and expansion pf the passive plate surface. The crosswise deformation per unit Voltage commencing from the bimorph is higher than the piezoelectric material. Bimorphs consist of two or more piezoelectric layers for construction. An anamorph i.e., Single piezo electric is used for study purpose, while the term is not commonly used (Kusculuoglu et al., 2004). Two classification are available and they are the two-electrode bimorph and the three-electrode bimorph as in Figure 5.

Figure 5 Configuration (see online version for colours)



When serial type is associated, one out of two ceramic plates is employed opposite to the direction of the polarisation. In this case to get rid of the polarised effect, the electric filed employed is limited. These kinds' serial bimorph benders are mostly used for sensing force and acceleration. However, the electrode parallel bimorph is being used (Zhang et al., 2014).

2 Problem solution techniques

2.1 Literature

Lot of research work has been completed in Cartesian domain to find and enhance the conduct multilayered piezoelectric bimorphs. Two dimensions cantilever beam has thoroughly studied (Her and Chen, 2018). The comparison lists a three-dimensional analytical analysis of rectangular, simply supported, multi-layer, laminated piezoelectric/passive plate (Ray et al., 1993). Investigations has carried out for numerically SS (simply supported) plates using a FE formulation, with various piezoelectric patch types (Shah et al., 1993). When piezoelectric patches are envisioned the dampen vibrations, the ideal positioning of the patches is at the places of highest displacement in free vibration of the plate (Batra et al., 1996). Thin plate theory is used to find finest actuator to plate thickness ratios for various Young modulus ratios by enhance the moment applied by a double layer piezoelectric actuator.

To determining the optimum thickness ratio and affecting the external applied forces on results are studies (Chaudhry and Rogers, 1994). And also, often estimate finest length ratios on a rectangular plate with fixed edges for a rectangular actuator area, the edge fixed condition being analogous, outside to an applied moment. Based on Cartesian axis, the above analyses are dynamic to the thoughtful of bimorph performance not able to apply to the circular case. Circular bimorphs domain has been analysed by at least two studies. A FE (finite element) system for flat, axisymmetric plates is also studied (Dobrucki and Pruchnicki, 1997). For the displacement of a bimorph an analytical solution is given for and also clasps the piezoelectric. The study also suggests free edges and a symmetric, dual-layer actuator. To boost the radius of a piezoelectric resonating actuator by increasing the thickness ratio studied (Hao et al., 2017). The effect of deflection on the micro cantilever beam is performed to optimise the different parameters such as length, thickness and the material's modulus by Young. The shape of the micro cantilever beam is varied, and the modified micro cantilever beam is found to provide improved micro cantilever beam deflection (Nasedkin, 2017). In this work, a circular shape piezoelectric with bimorph actuator is numerically analysed with various thickness and material of actuator and suitable thickness and material are found out to improve deformation of the piezoelectric actuator.

2.2 *Problem definition*

Advanced analyses such as piezoelectric effect can able to overcome the insufficient energy requirement in the on-flight conditions are aided by induced effects. In the view of predicting the complicated analysis behaviour, FEA is the only flexible methodology. In this simulation, a piezoelectric bimorph with a base membrane is selected for analysis. The bimorph is of annular shape and has 2 mm inner diameter and 9 mm outer diameter. The base membrane is of 1 mm thick and 10 mm diameter. The objectives of performing this piezoelectric simulation in ANSYS are

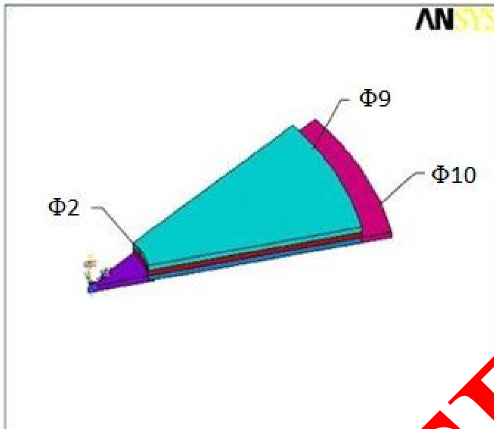
- to find the best material for the base membrane. Three most commonly used materials are selected for the analysis, namely Aluminium, Copper, and Silicon
- To find the optimum piezoelectric bimorph thickness for maximum deflection for a particular applied voltage of 200 volts.

3 ANSYS piezoelectric simulation

3.1 *Physical model*

The global *y*-coordinate is perpendicular to the plane of the circular bimorph and the base membrane combination. In ANSYS, the active *z*-coordinate is the direction of poling of the piezoelectric element mesh generated (Figure 6). In order to create a series bimorph, two local coordinate systems are generated with the local z-axis aligned with the global positively direction for one local coordinate system and with the local z-axis aligned with the global negative *y*-direction for the other local coordinate system. One volume is created in each of these two local coordinate systems to represent the top and bottom piezoelectric patch respectively. Since the problem is symmetric, a 30° sector is chosen for the analysis. Correspondingly proper symmetric boundary conditions are applied on the periphery of the volume mesh (Hagood et al., 990). The outer periphery of the base membrane is fixed against all translations (UX, UY, and UZ). Refer Figure (DeVoe and Pisano, 1997). Then a voltage (200 Volts) is applied between the top surface of the top piezo patch and the bottom surface of the bottom piezo patch. The piezoelectric bimorph with base membrane deflects the applied voltage along the globally positive *y*-direction. If the polarity of the voltage applied is reversed the deflection takes place in the opposite direction (Soloviev et al., 2018).

Figure 6 The geometry of the circular piezoelectric bimorph actuator (see online version for colours)



3.2 Finite element model

In this ANSYS program, the element SOLID226 has been carried for both the cases out of many 2D and 3D elements available for piezoelectric analysis (Roshani et al., 2018). Possible piezoelectric analyses available in the ANSYS APDL are static, modal, pre-stressed modal, harmonic, pre-stressed harmonic, and transient (Xu and Koko, 2004).

The SOLID226 elements with its degenerated shapes are shown in Figure 7. The individual element has 20 nodes with up to five degrees of freedom per node (UX, UY, UZ, TEMP, and VOLT). Figure 8 shows the structured meshed view of the piezoelectric bimorph with the base membrane.

Figure 7 SOLID226 piezoelectric element

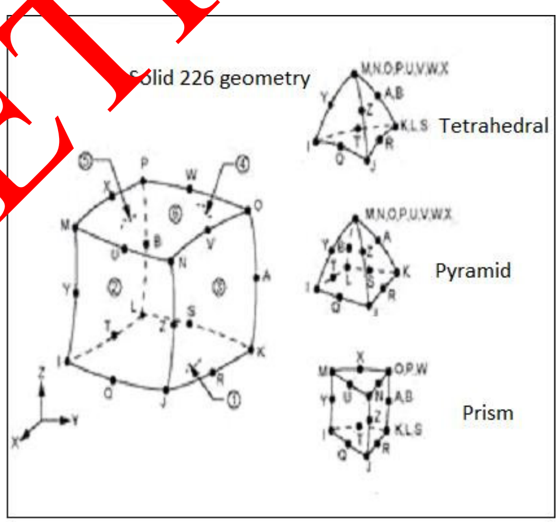
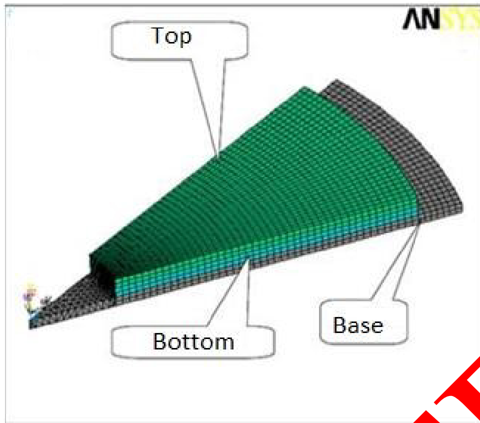


Figure 8 The 3D mesh of the piezoelectric bimorph with base membrane (see online version for colours)



3.3 Boundary conditions

In general boundary conditions are the major source to initiate the problem, especially the perfect boundary conditions can able to provide acceptable numerical simulation results. Figure 9 reveals the basic boundary condition applied in the circular piezoelectric bimorph with base membrane, in which fixed boundary conditions has been given to the outer part of the circular piezoelectric bimorph, and the reference model of this paper geometrically comes under symmetric conditions so 1/4th of the real model only taken for the analysis and thereby both sides of the model is given symmetric boundary condition

Figure 9 Boundary conditions of piezoelectric bimorph actuator (see online version for colours)

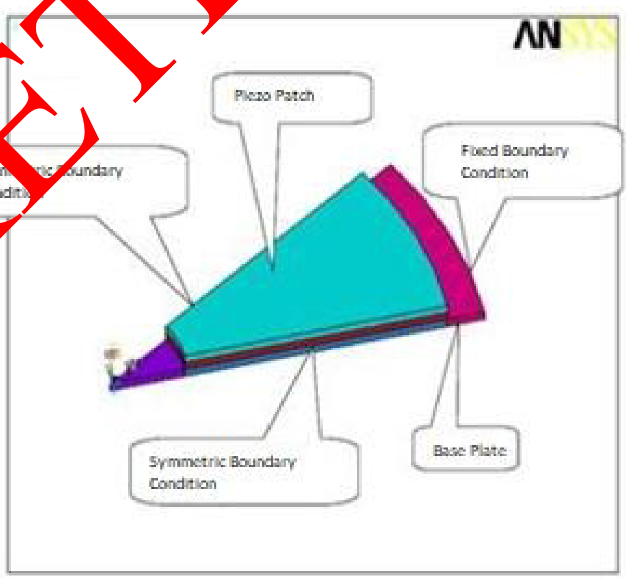


Figure 10 shows the voltages applied to the top and bottom piezoelectric patches.

Figure 10 The voltage applied to the top and bottom piezoelectric patch (see online version for colours)

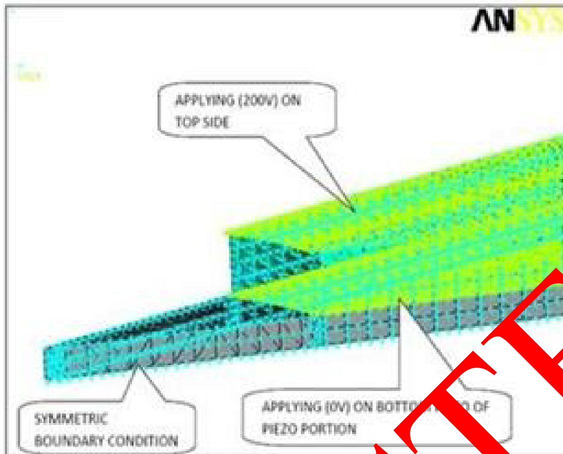


Table 1 provides the complete details of the mechanical properties of the three different materials which are assigned to the base membrane of the model

Table 1 Mechanical properties of the membrane

Material	*Young's Modulus* (GPa)	*Poisson's ratio*	*Density* (kg/m^3)
Aluminium	68.3	0.34	2689.8
Copper	[illegible]	0.34	7764
Silicon	112	0.28	2330

Table 2 provides the complete details of the mechanical properties of the circular piezoelectric bimorph.

Table 2 Mechanical properties of PZT

Parameter	*Symbols*	*Values*
Density (kg/m^3)	ρ	7800
Loss tangent (10^{-12} m^2/N)	$\tan\delta$	0.026
Anisotropic elasticity (10^{-12} m^2/N)	D_{11}	15.0
	$D_{22} = D_{33}$	19.0
	$D_{12} = D_{21}$	–4.50
	$D_{13} = D_{31}$	–5.70
	$D_{23} = D_{32}$	–5.70
	$D_{44} = D_{55}$	39.0
	D_{66}	49.4

Table 2 Mechanical properties of PZT (continued)

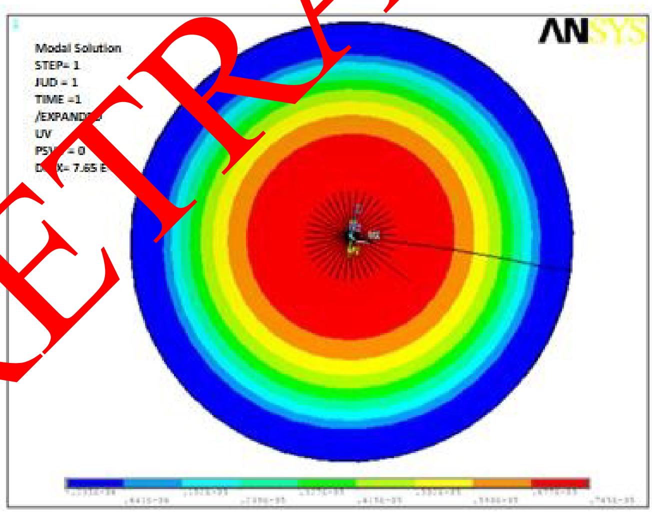
Parameters	*Symbols*	*Values*
Electric Permittivity (10^{-8} F/m)	ε_{11}^{T}	1.75
	ε_{22}^{T}	1.75
	ε^{T}	2.12
Piezoelectric matrix (Piezoelectic strain coefficients) ($10^{-1\circ}$ m/V)	d_{31}	–2.10
	d_{32}	–2.10
	d_{33}	5.0
	d_{24}	5.80
	d_{15}	5.80

4 Results

4.1 *Structural analysis*

Figure 11 shows the deflected shape of the piezoelectric bimorph with the base membrane, in which the maximum deflection occurred at the centre of the PZT bimorph actuator and the minimum displacement, is occurred at the outer part of the PZT

Figure 11 Deflected shape of the piezoelectric bimorph with the base membrane (see online version for colours)



A typical isometric view of the PZT bimorph actuator deflection has been shown in Figures 12 and 13.

Figure 12 Isometric view of the deflected shape (see online version for colours)

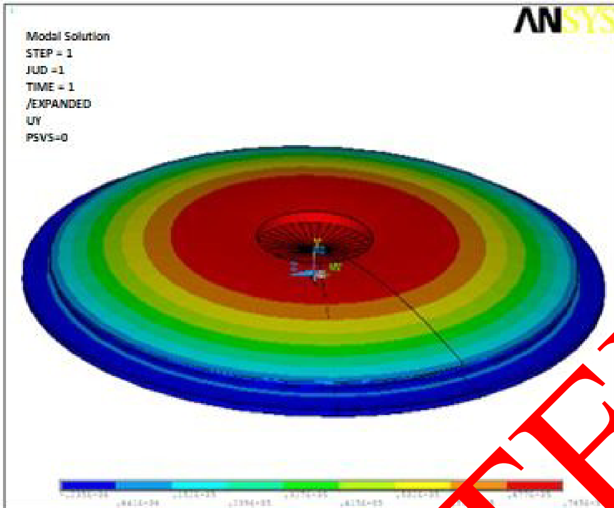
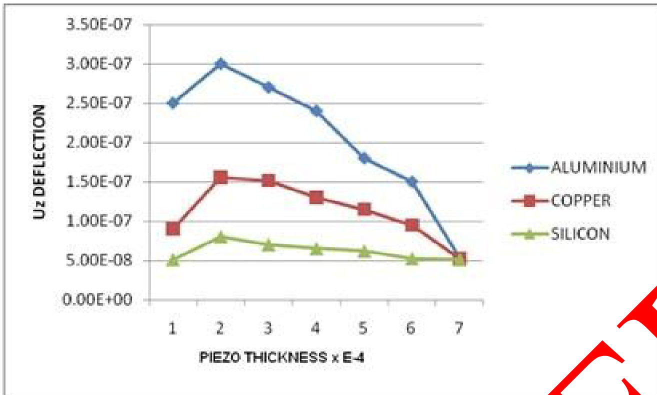


Figure 13 Deflection curve along a diametral line (see online version for colours)

4.2 Comparative result analysis

In this paper, comparative structural analyses have been carried out, in which the analyses comprise three materials with different thickness, varies from 1×10^{-4} m to 7×10^{-4} m. The overall deflection results in the z-direction for different thickness have been revealed in Figure 14. In this paper inverse piezoelectric effect, in which the voltage value has been given as constant input and then the corresponding deflections are noted for all the materials. In piezoelectric effect, high force and thereby high deflection can able to provide more electricity hence the comparative analyses mainly targeted the deflections output in order to predict the suitable material for PZT and its design.

Figure 14 Comparative results UZ(m) vs piezo thickness(10^{-4}m) for three materials (see online version for colours)

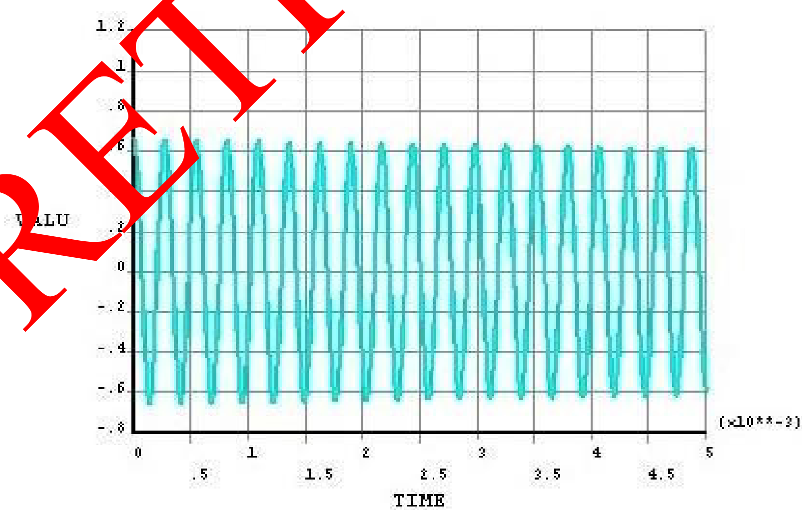


At piezo-thickness 7×10^{-4} m, all the deflections are the same for the given boundary conditions and at the piezo thickness of 1×10^{-4} m; the induced deflection is quite high in the aluminium compared to the other two materials

4.3 Transient analysis

To predict the damping ratio, transient analysis is carried out and the stability for the circular piezoelectric bimorph while undergoes load. The resulting plot has been shown in Figure 15, in which the fundamental data for damping estimation have been estimated. Number of cycles considered as 17, Load applied is 0.2424 N, overall time process is 5×10^{-3} s, Deltime 1×10^{-5} s, $X_1 = 0.661645$ mm and $X_n + 1 = 0.612988$ mm.

Figure 15 Transient analysis result (see online version for colours)



By logarithmic decrement method, we know logarithmic decrement

$$\delta = \frac{1}{n}\log_e \frac{X_1}{X_{n+1}} \tag{1}$$

where

n: number of cycles considered

X_1: maximum amplitude in the first cycle

X_{n+1}: Maximum amplitude in $n + 1$ cycle.

Substituting the values in equation (1) and then

$$\delta = 4.493165 \times 10^{-3}$$

We also know that

$$\delta = \frac{2\pi\varsigma}{\sqrt{1-\varsigma}} \tag{2}$$

$$\varsigma^2 = \frac{\delta^2}{4\pi^2 + \delta^2} \tag{3}$$

where ζ = Damping ratio

Substituting the values in equation (3) and then

$$\zeta = 7.15109 \times 10^{-4}$$

The value of the damping ratio for circular piezoelectric bimorph with the membrane is below one so the system is underdamped, which means after the load applied the reference model undergoes long settling time vibration.

5 Conclusion

The finite element analysis is carried out for circular piezoelectric bimorph with the help of Mechanical APDL Product Launcher. The effect of thickness on deformation is studied by the varying thickness and it is found that circular piezoelectric bimorph with minimum membrane thickness provides a better result. The computational results can match with the theoretical result, but the losses have to incorporate in their theoretical formula for accuracy. The three different materials such as aluminium, copper, and silicon are taken into consideration and aluminium has significant improvement in deformation of the circular piezoelectric bimorph. Hence it is found that the deformation of the membrane is proportional to the drive voltage and frequency.

References

Batra, R.C., Liang, X.Q. and Yang, J.S. (1996) 'Shape control of vibrating simply supported rectangular plates', *AIAA Journal*, Vol. 34, No. 1, pp.116–122.

Bisegna, P. and Caruso, G. (2001) 'Evaluation of higher-order theories of piezoelectric plates in bending and in stretching', *Int. J. Solids Struct.*, Vol. 38, pp.8805–8830.

Chaudhry, Z. and Rogers, C.A. (1994) 'Performance and optimization of induced strain actuated structures under external loading', *AIAA Journal*, Vol. 32, No. 6, pp.1289–1294.

DeVoe, D.L. and Pisano, A.P. (1997) 'Modeling and optimal design of piezoelectric cantilever microactuators', *J. Micro. Electro. Mech.*, Vol. 6, No. 3, pp.266–270.

Dobrucki, A.B. and Pruchnicki, P. (1997) 'Theory of piezoelectric axisymmetric bimorph', *Sensors Actuators A*, Vol. 58, pp.203–212.

Hagood, N.W., Chung, W.H. and Von Flotow, A. (1990) 'A von flotow, modelling of piezoelectric actuator dynamics for active structural control', *J. Intell. Mater. Syst. Struct.*, Vol. 1, pp.327–354.

Hao, H., Jenkins, K., Huang, X., Xu, Y., Huang, J. and Yang, R. (2017) 'Piezoelectric potential in single-crystalline ZNO Nano helices based on finite element analysis', *Nanomaterials*, Vol. 7, p.430.

He, L-H., Lim, C.W. and Soh, A.K. (2000) 'Three-dimensional analysis of an antiparallel piezoelectric bimorph', *Acta Mech.*, Vol. 145, pp.189–204.

Her, S-C. and Chen, H-Y. (2018) 'Stress analysis of sandwich composite beam induced by piezoelectric layer', *Journal of Applied Biomaterials & Functional Materials*, Vol. 16, No. 1_suppl., 4 April, pp.132–139.

Kim, S.J. and Jones, J.D. (1991) 'Optimal design of piezo actuators for active noise and vibration control', *AIAA Journal*, Vol. 29, No. 12, pp.2047–2053.

Kusculuoglu, Z.K., Fallahi, B. and Royston, T.J. (2004) 'Finite element model of a beam with a piezoceramic patch actuator', *J. Sound Vib.*, Vol. 276, pp.27–44.

Mathew Alphonse, N.N. and Ramesh Kumar, R. (2017) 'Shape modification of microcantilever beam for biosensing applications', Research Journal of Pharmaceutical, Biological and Chemical Sciences, ISSN: 0975-8585, Vol. 7, No. [illegible], pp.1398–1403.

Mu, Y.H., Hung, N.P. and Ngoi, K.A. ([illegible]) 'Optimization design of micropump', *Int. J. Adv. Manuf. Technol.*, Vol. 15, pp.573–576.

Murugu Nachippan, N., Balaji, V., Subba Reddy, S.V. and Logesh, K. (2015) 'Enhancement of deflection of microcantilever beam for improving the sensitivity of biosensor', *International Journal of ChemTech Research CODEN (USA): IJCRGG*, Vol. 8, No. 8, pp.349–356, ISSN: 0974-4290.

Nasedkin, A.V. (2017) Finite Element Simulation of Dissipative Heating of Piezoelectric Vibratory Gyroscopes, Advanced Materials, Springer Proceedings in Physics, Vol. 193, Springer, Cham.

Ray, M.Ch., Rao, K.M. and Samanta, B. (1993) 'Exact solutions for static analysis of intelligent structures', *AIAA Journal*, Vol. 31, No. 9, pp.1684–1691.

Roshani, H., [illegible], S., Montoya, A., Papagiannakis, A.T. and Abba, A. (2018) 'Development and *Finite Element Analysis of Piezoelectric-Based Prototypes for Harvesting Energy From Roadway Pavement*, Advancement in the Design and Performance of Sustainable Asphalt Pavements, GeoMEast 2017 Sustainable Civil Infrastructures, Springer, Cham.

Shah, D.K., Joshi, S.P. and Chan, W.S. (1993) 'The Static structural response of plates with piezoceramic layers', *Smart Mater. Struct.*, Vol. 2, pp.172–180.

Smits, J.G. and Ballato, A. (1994) 'Dynamic admittance of piezoelectric cantilever bimorphs', *J. Micro Electro. Mech. Syst.*, Vol. 3, No. 3, pp.105–112.

Soloviev, A.N., Parinov, I.A., Cherpakov, A.V., Chebanenko, V.A., Rozhkov, E.V. and Duong, L.V. (2018) *Analysis of the Performance of the Cantilever-type Piezoelectric Generator Based on Finite Element Modelling*, Advances in Structural Integrity, Springer, Singapore.

Xu, S.X. and Koko, T.S. (2004) 'Finite element analysis and design of actively controlled piezoelectric smart structures', *Finite Elements Anal. Design*, Vol. 40, pp.241–262.

Zhang, A.G., Yang, T.J., Du, J.T., Lv, P. and Li, X.G. (2014) 'Finite element analysis of piezoelectric materials', *Advanced Materials Research*, Vols. 860–863, pp.872–875.